Iranian Journal of Mathematical Sciences and Informatics
Vol. 12, No. 2 (2017), pp 101-109.
DOI: 10.7508/ijmsi.2017.2.006

Inequalities for the Derivatives of a Polynomial

Abdullah Mir[a,*], Q. M. Dawood[a] and Bilal Dar[b]

[a]Department of Mathematics, University of Kashmir,
Srinagar, 190006, India.
[b]Govt. Degree College (Boys), Pulwama-192301.

E-mail: mabdullah_mir@yahoo.co.in
E-mail: qdawood@gmail.com
E-mail: darbilal85@ymail.com

Abstract. The paper presents an L^r- analogue of an inequality regarding the s^{th} derivative of a polynomial having zeros outside a circle of arbitrary radius but greater or equal to one. Our result provides improvements and generalizations of some well-known polynomial inequalities.

Keywords: Polynomial, Zeros, s^{th} Derivative.
2010 Mathematics subject classification: 30A10, 30C10, 30C15.

1. Introduction and Statement of Results

Let $P(z)$ be a polynomial of degree at most n and $P'(z)$ be its derivative, then

$$\max_{|z|=1}|P'(z)| \leq n \max_{|z|=1}|P(z)| \tag{1.1}$$

*Corresponding Author

Received 26 February 2015; Accepted 05 December 2015
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[DOI: 10.7508/ijmsi.2017.2.006]

and for every $r \geq 1$,

$$\left\{ \int_0^{2\pi} |P'(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}} \leq n \left\{ \int_0^{2\pi} |P(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}}. \tag{1.2}$$

Inequality (1.1) is a classical result of Bernstein[6] whereas inequality (1.2) is due to Zygmund[15] who proved it for all trigonometric polynomials of degree n and not only for those which are of the form $P(e^{i\theta})$. Arestov[1] proved that (1.2) remains true for $0 < r < 1$ as well. If $r \to \infty$ in inequality (1.2), we get (1.1).

If we restrict ourselves to the class of polynomials having no zeros in $|z| < 1$, then both the inequalities (1.1) and (1.2) can be sharpened. In fact, If $P(z) \neq 0$ in $|z| < 1$, then (1.1) and (1.2) can be respectively replaced by

$$\max_{|z|=1} |P'(z)| \leq \frac{n}{2} \max_{|z|=1} |P(z)| \tag{1.3}$$

and

$$\left\{ \int_0^{2\pi} |P'(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}} \leq nA_r \left\{ \int_0^{2\pi} |P(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}}, \tag{1.4}$$

where $A_r = \left\{ \frac{1}{2\pi} \int_0^{2\pi} |1 + e^{i\alpha}|^r d\alpha \right\}^{\frac{-1}{r}}$.

Inequality (1.3) was conjectured by Erdös and later verified by Lax[11], whereas inequality (1.4) was proved by De-Bruijn[7] for $r \geq 1$. Rahman and Schemeisser[13] later proved that (1.4) holds for $0 < r < 1$ also. If $r \to \infty$ in (1.4), we get (1.3).

As a generalization of (1.3) Malik[12] proved that if $P(z) \neq 0$ in $|z| < k,\ k \geq 1$, then

$$\max_{|z|=1} |P'(z)| \leq \frac{n}{1+k} \max_{|z|=1} |P(z)|, \tag{1.5}$$

whereas under the same hypothesis, Govil and Rahman[9] extended inequality (1.4) by showing that

$$\left\{ \int_0^{2\pi} |P'(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}} \leq nE_r \left\{ \int_0^{2\pi} |P(e^{i\theta})|^r d\theta \right\}^{\frac{1}{r}}, \tag{1.6}$$

where $E_r = \left\{ \frac{1}{2\pi} \int_0^{2\pi} |k + e^{i\alpha}|^r d\alpha \right\}^{\frac{-1}{r}},\ r \geq 1.$

In the same paper, Govil and Rahman[9, Theorem 4] extended inequality (1.5) to the s^{th} derivative of a polynomial and proved under the same hypothesis

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[DOI: 10.7508/ijmsi.2017.2.006]

for $1 \leq s < n$ that

$$\max_{|z|=1} |P^{(s)}(z)| \leq \frac{n(n-1)\cdots(n-s+1)}{1+k^s} \max_{|z|=1} |P(z)|. \tag{1.7}$$

Inequality (1.7) was refined by Aziz and Rather [3, Corollary 1] by involving the binomial coefficients $C(n,s),\ 1 \leq s < n$ and coefficients of the polynomial $P(z)$. In fact they proved that if $P(z) = \sum_{j=0}^{n} a_j z^j$ does not vanish in $|z| < k,\ k \geq 1$, then for $1 \leq s < n$,

$$\max_{|z|=1} |P^{(s)}(z)| \leq \frac{n(n-1)\cdots(n-s+1)}{1+\psi_{k,s}} \max_{|z|=1} |P(z)|, \tag{1.8}$$

where

$$\psi_{k,s} = k^{s+1}\left(\frac{1+\frac{1}{C(n,s)}\left|\frac{a_s}{a_0}\right|k^{s-1}}{1+\frac{1}{C(n,s)}\left|\frac{a_s}{a_0}\right|k^{s+1}}\right). \tag{1.9}$$

In the literature there exist various results regarding the estimates for polynomials and for general analytic functions and also the approximations of polynomials and their derivatives (for example see[8],[14]). In this paper, we prove the following result which refines the inequality (1.8).

Theorem 1.1. *If $P(z) = \sum_{j=0}^{n} a_j z^j$ is a polynomial of degree n having no zeros in $|z| < k,\ k \geq 1$, and $m = \min_{|z|=k} |P(z)|$ then for $1 \leq s < n$,*

$$\max_{|z|=1} |P^{(s)}(z)| \leq \frac{n(n-1)\cdots(n-s+1)}{1+\psi_{k,s}} \Big(\max_{|z|=1} |P(z)| - \frac{m\psi_{k,s}}{k^n} \Big), \tag{1.10}$$

where $\psi_{k,s}$ is defined by (1.9).

The result is best possible for $k = 1$ and equality holds for $P(z) = z^n + 1$.

Remark 1.2. For $s = 1$ and $m = 0$, Theorem 1.1 reduces to a result of Govil et. al.[10, Theorem 1] and for $k = s = 1$, inequality (1.10) reduces to a result of Aziz and Dawood[2, Theorem A].

Remark 1.3. Note by inequality (2.2) of Lemma 2.1 (stated in section 2) that $\frac{1}{C(n,s)}\left|\frac{a_s}{a_0}\right|k^s \leq 1$, which can easily be shown to be equivalent to $\psi_{k,s} \geq k^s,\ 1 \leq s < n$. Using this fact in inequality (1.10), we get the following improvement of inequality (1.7).

Corollary 1.4. *If $P(z) = \sum_{j=0}^{n} a_j z^j$ is a polynomial of degree n having no zeros in $|z| < k,\ k \geq 1$, and $m = \min_{|z|=k} |P(z)|$ then for $1 \leq s < n$,*

$$\max_{|z|=1} |P^{(s)}(z)| \leq \frac{n(n-1)\cdots(n-s+1)}{1+k^s} \Big(\max_{|z|=1} |P(z)| - \frac{m}{k^{n-s}} \Big). \tag{1.11}$$

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In order to prove the Theorem 1.1, we prove the following more general result which extends Theorem 1.1 to its corresponding L^r- analogue.

Theorem 1.5. *If $P(z)=\sum\limits_{j=0}^{n}a_jz^j$ is a polynomial of degree n having no zeros in $|z|<k,\ k\geq 1$, and $m=\min_{|z|=k}|P(z)|$, then for every complex number β with $|\beta|\leq 1$ and $1\leq s<n$, we have*

$$\left\{\int\limits_0^{2\pi}\left|P^{(s)}(e^{i\theta})+\frac{\beta mn(n-1)\cdots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right|^r d\theta\right\}^{\frac{1}{r}}$$

$$\leq n(n-1)\cdots(n-s+1)C_r\left\{\int\limits_0^{2\pi}|P(e^{i\theta})|^r d\theta\right\}^{\frac{1}{r}}, \qquad (1.12)$$

where $C_r=\left\{\frac{1}{2\pi}\int\limits_0^{2\pi}|\psi_{k,s}+e^{i\alpha}|^r d\alpha\right\}^{\frac{-1}{r}}$, $r>0$ and $\psi_{k,s}$ is defined by (1.9).

Remark 1.6. Using the fact that $\psi_{k,s}\geq k^s$ and take $\beta=0$ in inequality (1.12), we obtain a result of Aziz and Shah[5].

2. Lemmas

We need the following lemmas for the proofs of Theorems. Here, throughout this paper we write $Q(z)=z^n\overline{P(\frac{1}{\bar{z}})}$.

Lemma 2.1. *If $P(z)=\sum\limits_{j=0}^{n}a_jz^j$ is a polynomial of degree n which does not vanish in $|z|<k, k\geq 1$, then for $1\leq s<n$ and $|z|=1$,*

$$\left|Q^{(s)}(z)\right|\geq\psi_{k,s}\left|P^{(s)}(z)\right|, \qquad (2.1)$$

and

$$\frac{1}{C(n,s)}\left|\frac{a_s}{a_0}\right|k^s\leq 1, \qquad (2.2)$$

where $\psi_{k,s}$ is defined by (1.9).

The above lemma is due to Aziz and Rather[3].

Lemma 2.2. *If $P(z)$ is a polynomial of degree n, then for each $\alpha,\ 0\leq\alpha<2\pi$ and $r>0$, we have*

$$\int\limits_0^{2\pi}\int\limits_0^{2\pi}\left|Q'(e^{i\theta})+e^{i\alpha}P'(e^{i\theta})\right|^r d\theta d\alpha\leq 2\pi n^r\int\limits_0^{2\pi}|P(e^{i\theta})|^r d\theta. \qquad (2.3)$$

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The above lemma is due to Aziz and Shah[4].

Lemma 2.3. *If* $P(z) = \sum_{j=0}^{n} a_j z^j$ *is a polynomial of degree* n *which does not vanish in* $|z| < k, k \geq 1$*, then for* $1 \leq s < n$ *and* $|z| = 1$*,*

$$\left|Q^{(s)}(z)\right| \geq \psi_{k,s}\left|P^{(s)}(z)\right| + \frac{mn(n-1)\cdots(n-s+1)}{k^n}\psi_{k,s}, \tag{2.4}$$

where $m = \min_{|z|=k} |P(z)|$.

Proof. Since $m \leq |P(z)|$ for $|z| = k$, we have for every β with $|\beta| < 1$,

$$\left|\frac{m\beta z^n}{k^n}\right| < |P(z)| \ for \ |z| = k.$$

Therefore by Rouche's theorem $P(z) + \frac{m\beta z^n}{k^n}$ has no zero in $|z| < k,\ k \geq 1$. Applying Lemma 2.1 to the polynomial $P(z) + \frac{m\beta z^n}{k^n}$, we get for $1 \leq s < n$ and $|z| = 1$,

$$\left|Q^{(s)}(z)\right| \geq \psi_{k,s}\left|P^{(s)}(z) + \frac{mn(n-1)\cdots(n-s+1)\beta}{k^n}\right|. \tag{2.5}$$

Choose the argument of β so that

$$\left|P^{(s)}(z) + \frac{mn(n-1)\cdots(n-s+1)\beta z^{n-s}}{k^n}\right| = \left|P^{(s)}(z)\right| + \frac{mn(n-1)\cdots(n-s+1)|\beta z^{n-s}|}{k^n},$$

it follows from (2.5) that for $|z| = 1$,

$$\left|Q^{(s)}(z)\right| \geq \psi_{k,s}\left|P^{(s)}(z)\right| + \frac{mn(n-1)\cdots(n-s+1)|\beta z^{n-s}|}{k^n}\psi_{k,s}. \tag{2.6}$$

Letting $|\beta| \to 1$ in inequality (2.6), we get

$$\left|Q^{(s)}(z)\right| \geq \psi_{k,s}\left|P^{(s)}(z)\right| + \frac{mn(n-1)\cdots(n-s+1)}{k^n}\psi_{k,s}.$$

This completes the proof of Lemma 2.3. □

Lemma 2.4. *If* A, B, C *are non-negative real numbers such that* $B + C \leq A$*. Then for every real* α*,*

$$\left|(A - C) + e^{i\alpha}(B + C)\right| \leq \left|A + e^{i\alpha}B\right|. \tag{2.7}$$

The above lemma is due to Aziz and Shah[4].

3. Proofs of Theorems

Proof of the Theorem 1.5. Since $P(z)$ is a polynomial of degree n , $P(z) \neq 0$ in $|z| < k, k \geq 1$, and $Q(z) = z^n\overline{P(\frac{1}{\bar{z}})}$. Therefore, for each $\alpha, 0 \leq \alpha < 2\pi, F(z) = Q(z) + e^{i\alpha}P(z)$ is a polynomial of degree n and we have

$$F^{(s)}(z) = Q^{(s)}(z) + e^{i\alpha}P^{(s)}(z),$$

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which is clearly a polynomial of degree $n-s, 1 \leq s < n$. By the repeated application of inequality (1.2), we have for each $r > 0$,

$$\begin{aligned}
&\int_0^{2\pi} \left|Q^{(s)}(e^{i\theta}) + e^{i\alpha}P^{(s)}(e^{i\theta})\right|^r d\theta \\
&\qquad \leq (n-s+1)^r \int_0^{2\pi} \left|Q^{(s-1)}(e^{i\theta}) + e^{i\alpha}P^{(s-1)}(e^{i\theta})\right|^r d\theta \\
&\qquad \leq (n-s+1)^r(n-s+2)^r \int_0^{2\pi} \left|Q^{(s-2)}(e^{i\theta}) + e^{i\alpha}P^{(s-2)}(e^{i\theta})\right|^r d\theta \\
&\qquad . \\
&\qquad . \\
&\qquad . \\
&\qquad \leq (n-s+1)^r(n-s+2)^r \ldots (n-1)^r \int_0^{2\pi} \left|Q'(e^{i\theta}) + e^{i\alpha}P'(e^{i\theta})\right|^r d\theta.
\end{aligned} \tag{3.1}$$

Integrating inequality (3.1) with respect to α over $[0, 2\pi]$ and using inequality (2.3) of Lemma 2.2, we get

$$\begin{aligned}
&\int_0^{2\pi}\int_0^{2\pi} \left|Q^{(s)}(e^{i\theta}) + e^{i\alpha}P^{(s)}(e^{i\theta})\right|^r d\theta d\alpha \\
&\leq 2\pi(n-s+1)^r(n-s+2)^r \ldots (n-1)^r n^r \int_0^{2\pi} \left|P(e^{i\theta})\right|^r d\theta.
\end{aligned} \tag{3.2}$$

Now, from inequality (2.4) of Lemma 2.3, it easily follows that

$$\begin{aligned}
&\psi_{k,s}\left\{\left|P^{(s)}(e^{i\theta})\right| + \frac{mn(n-1)\ldots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right\} \\
&\qquad \leq \left|Q^{(s)}(e^{i\theta})\right| - \frac{mn(n-1)\ldots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}.
\end{aligned} \tag{3.3}$$

Taking $A = \left|Q^{(s)}(e^{i\theta})\right|, B = \left|P^{(s)}(e^{i\theta})\right|, C = \frac{mn(n-1)\ldots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}$ and noting that $\psi_{k,s} \geq k^s \geq 1, 1 \leq s < n$, so that by (3.3),

$$B + C \leq \psi_{k,s}(B + C) \leq A - C \leq A,$$

we get from Lemma 2.4 that

$$\begin{aligned}
&\left|\left\{\left|Q^{(s)}(e^{i\theta})\right| - \frac{mn(n-1)\ldots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right\}\right. \\
&\qquad \left. + e^{i\alpha}\left\{\left|P^{(s)}(e^{i\theta})\right| + \frac{mn(n-1)\ldots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right\}\right| \\
&\qquad \leq \left|\left|Q^{(s)}(e^{i\theta})\right| + e^{i\alpha}\left|P^{(s)}(e^{i\theta})\right|\right|.
\end{aligned}$$

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This implies for each $r > 0$,

$$\int_0^{2\pi} \left|F(\theta) + e^{i\alpha}G(\theta)\right|^r d\alpha \leq \int_0^{2\pi} \left|\left|Q^{(s)}(e^{i\theta})\right| + e^{i\alpha}\left|P^{(s)}(e^{i\theta})\right|\right|^r d\alpha, \quad (3.4)$$

where

$$F(\theta) = \left|Q^{(s)}(e^{i\theta})\right| - \frac{mn(n-1)\dots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}$$

and

$$G(\theta) = \left|P^{(s)}(e^{i\theta})\right| + \frac{mn(n-1)\dots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}.$$

Integrating inequality (3.4) with respect to θ on $[0, 2\pi]$ and using inequality (3.2), we obtain

$$\begin{aligned}
\frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} &\left|F(\theta) + e^{i\alpha}G(\theta)\right|^r d\alpha d\theta \\
&\leq \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} \left|\left|Q^{(s)}(e^{i\theta})\right| + e^{i\alpha}\left|P^{(s)}(e^{i\theta})\right|\right|^r d\alpha d\theta \\
&= \frac{1}{2\pi}\int_0^{2\pi}\int_0^{2\pi} \left|Q^{(s)}(e^{i\theta}) + e^{i\alpha}P^{(s)}(e^{i\theta})\right|^r d\alpha d\theta \\
&\leq (n-s+1)^r(n-s+2)^r\dots(n-1)^r n^r \int_0^{2\pi} \left|P(e^{i\theta})\right|^r d\theta.
\end{aligned} \quad (3.5)$$

Now for every real number α and $t_1 \geq t_2 \geq 1$, we have

$$|t_1 + e^{i\alpha}| \geq |t_2 + e^{i\alpha}|,$$

which implies for every $r > 0$,

$$\int_0^{2\pi} |t_1 + e^{i\alpha}|^r d\alpha \geq \int_0^{2\pi} |t_2 + e^{i\alpha}|^r d\alpha.$$

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[DOI: 10.7508/ijmsi.2017.2.006]

If $G(\theta) \neq 0$, we take $t_1 = \left|\frac{F(\theta)}{G(\theta)}\right|$ and $t_2 = \psi_{k,s}$, then from (3.3) and noting that $\psi_{k,s} \geq 1$, we have $t_1 \geq t_2 \geq 1$, hence

$$\begin{aligned}\int_0^{2\pi} \left|F(\theta) + e^{i\alpha}G(\theta)\right|^r d\alpha &= |G(\theta)|^r \int_0^{2\pi} \left|\frac{F(\theta)}{G(\theta)} + e^{i\alpha}\right|^r d\alpha \\ &= |G(\theta)|^r \int_0^{2\pi} \left|\left|\frac{F(\theta)}{G(\theta)}\right| + e^{i\alpha}\right|^r d\alpha \\ &\geq |G(\theta)|^r \int_0^{2\pi} \left|\psi_{k,s} + e^{i\alpha}\right|^r d\alpha \\ &= \left\{\left|P^{(s)}(e^{i\theta})\right| + \frac{mn(n-1)\dots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right\}^r \\ &\int_0^{2\pi} \left|\psi_{k,s} + e^{i\alpha}\right|^r d\alpha. \end{aligned} \tag{3.6}$$

For $G(\theta) = 0$, this inequality is trivially true. Using this in (3.5), it follows for each $r > 0$,

$$\begin{aligned}&\int_0^{2\pi} \left\{\left|P^{(s)}(e^{i\theta})\right| + \frac{mn(n-1)\dots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right\}^r d\theta \\ &\leq \frac{(n-s+1)^r(n-s+2)^r\dots(n-1)^r n^r}{\frac{1}{2\pi}\int_0^{2\pi}\left|\psi_{k,s} + e^{i\alpha}\right|^r d\alpha} \int_0^{2\pi} \left|P(e^{i\theta})\right|^r d\theta. \end{aligned} \tag{3.7}$$

Now using the fact that for every β with $|\beta| \leq 1$,

$$\left|P^{(s)}(e^{i\theta}) + \frac{\beta mn(n-1)\dots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})}\right| \leq \left|P^{(s)}(e^{i\theta})\right| + \frac{mn(n-1)\dots(n-s+1)\psi_{k,s}}{k^n(1+\psi_{k,s})},$$

the desired result follows from (3.7).

Proof of the Theorem 1.1 Making $r \to \infty$ and choosing the argument of β suitably with $|\beta| = 1$ in (1.12), Theorem 1.1 follows.

Acknowledgement

The authors are highly thankful to the referee for his valuable suggestions regarding the paper.

References

1. V. V. Arestov, On Integral Inequalities for Trigonometric Polynomials and Their Derivatives, *Izv. Akad. Nauk. SSSR. Ser. Mat.*, **45**, (1981), 3-22.
2. A. Aziz, Q. M. Dawood, Inequalities for a Polynomial and its Derivative, *J.Approx. Theory*, **54**, (1988), 306-313.
3. A. Aziz, N. A. Rather, Some Zygmund Type L^p Inequalities for Polynomials, *J.Math. Anal. Appl.*, **289**, (2004), 14-29.
4. A. Aziz, W. M. Shah, L^p Inequalities for Polynomials with Restricted Zeros, *Glas. Mate.*, **32**, (1997), 247-258.

[Downloaded from ijmsi.com on 2026-11-01]

[DOI: 10.7508/ijmsi.2017.2.006]

5. A. Aziz, W. M. Shah, L^p Inequalities for Polynomials with Restricted Zeros, *Proc. Indian Acad. Sci. (Math. Sci.)*, **108**, (1998), 63-68.
6. S. Bernstein, *Lecons Sur Les Proprietes Extremals et la Meilleure Approximation des Fonctions Analytiques d'une Fonctions Reelle*, Gauthier-villars Paris, 1926.
7. N. G. De-Bruijn, Inequalities Concerning Polynomials in the Complex Domain, *Nederl. Akad. Wetnesch Proc.*, **50**, (1947), 1265-127.
8. M. R. Eslahchi, Sonaz Amani, The Best Uniform Polynomial Approximation of Two Classes of Rational Functions, *Iranian Journal of Mathematical Sciences and Informatics*, **7**, (2012), 93-102.
9. N. K. Govil, , Q. I. Rahman, Functions of Exponential Type not Vanishing in a Half-Plane and Related Polynomials, *Trans. Amer. Math. Soc.*, **137**, (1969), 501-517.
10. N. K. Govil, Q. I. Rahman, G. Schemeisser, On the Derivative of a Polynomial, *Illinois J. Math.*, **23**, (1979), 319-330.
11. P. D. Lax, Proof of a Conjecture of P. Erdös on the Derivative of a Polynomial, *Bull. Amer. Math. Soc.*, **50**, (1944), 509-513.
12. M. A. Malik, On the Derivative of a Polynomial, *J. London Math. Soc.*, **1**, (1969), 57-60.
13. Q. I. Rahman, G. Schemeisser, L^p Inequalities for Polynomials, *J. Approx. Theory*, **53**, (1988), 26-32.
14. Rasoul Aghalary, Applications of the Norm Estimates for Univalance of Analytic Functions, *Iranian Journal of Mathematical Sciences and Informatics*, **9**, (2014), 101-108.
15. A. Zygmund, A Remark on Conjugate Series, *Proc. London Math. Soc.*, **34**, (1932), 392-400.

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[DOI: 10.7508/ijmsi.2017.2.006]