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On a Group of the Form $2^{4+5}{:}GL(4,2)$

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Abstract. The affine general linear group $2^5{:}GL(5,2)$ of $GL(6,2)$ has 6 conjugacy classes of maximal subgroups. The largest two maximal subgroups are of the forms $2_+^{1+8}{:}GL(4,2)$ and $2^{4+5}{:}GL(4,2)$. In this article we consider the group $2^{4+5}{:}GL(4,2)$, which we denote by $\overline{G}$. Firstly we determine its conjugacy classes using the coset analysis technique. The structures of the inertia factor groups are also determined. We then compute all the Fischer matrices and apply the Clifford-Fischer theory to compute the ordinary character table of $\overline{G}$. Using information on conjugacy classes, Fischer matrices and both ordinary and projective character tables of the inertia factor groups, we concluded that we need to use the ordinary character tables of all the inertia factor groups to construct the character table of $\overline{G}$. The character table of $\overline{G}$ is a 75×75 complex valued

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matrix and we supply it (in the format of Clifford-Fischer theory) at the end of this paper as Table 6.

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1. Introduction

Let $GL(n,q)$ be the finite general linear group consisting of $n \times n$ invertible matrices over $\mathbb{F}_q$. It is well-known that the affine general linear subgroup of $GL(n,q)$, denoted by $AGL(n,q)$, is a group of the form $q^{n-1}{:}GL(n-1,q)$. With the help of GAP [18], we were able to determine the structures of all the maximal subgroups of the affine general linear group $AGL(6,2) = 2^5{:}GL(5,2) := M$ of $GL(6,2)$. Representatives M_i of these maximal subgroups can be taken as follows:

M_i	$\lvert M_i \rvert$	$[M : M_i]$
$2_+^{1+8}{:}GL(4,2)$	10321920	31
$2^{4+5}{:}GL(4,2)$	10321920	31
$GL(5,2)$	9999360	32
$2^{2+9}{:}(GL(3,2) \times S_3)$	2064384	155
$2^9{:}(GL(3,2) \times S_4)$	2064384	155
$(2^5{:}31){:}5$	4960	64512

Table 1. The maximal subgroups of $M = AGL(6,2) = 2^5{:}GL(5,2)$.

The first group $M_1 = 2_+^{1+8}{:}GL(4,2)$ has been handled by the first two authors in [15] as a special case of an investigation on the general case $2_+^{1+2n}{:}GL(n,2)$, which was proved to exist for small values of n and conjectured by the authors to exist for all values of n. In this article we handle the other largest maximal subgroup of M, namely the group $2^{4+5}{:}GL(4,2) := \overline{G}$. This is a split extension of the special group 2^{4+5} by the linear group $GL(4,2) \cong A_8$. Note that the kernel of this extension is a 2-group of order 512 with center, Frattini and derived groups all are isomorphic to 2^4 and $2^{4+5}/2^4$ is an elementary abelian group of order 2^5. We firstly determine the conjugacy classes of $\overline{G}$ using the coset analysis technique. Then we determine the inertia factor groups and their fusions into $GL(4,2)$. We also calculate the Fischer matrices and hence the ordinary character table of $\overline{G}$ using the theory of Clifford-Fischer matrices. This is a very good example for the applications of Clifford-Fischer Theory since the kernel of the extension is a non-abelian group. Not many examples of this type have been studied via Clifford-Fischer Theory. In most of the papers in the literature, the

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kernel of the extension was either abelian or an extra special p-group. In our case here, the kernel is a special 2-group and consequently the whole structure of the extension becomes more complicated. For example many of the properties of the Fischer matrices mentioned in [9] or [15] can not be satisfied here. The Fischer matrices of $\overline{G}$ have all been determined in this paper and their sizes range between 2 and 12. The character table of $\overline{G}$ is a 75×75 complex valued matrix and is partitioned into six blocks corresponding to the six inertia factor groups $H_1 = H_2 = GL(4,2)$, $H_3 = H_4 = H_5 = 2^3{:}GL(3,2)$ and $H_6 = 2^4{:}D_{12}$ (see Section 3). If one was only interested in the calculation of the character table, then it could be computed by using GAP or Magma [16] and the generators $\overline{g}_1$ and $\overline{g}_2$ of $\overline{G}$, given below. But Clifford-Fischer Theory provides much more interesting and practical information on the group and on the character table, in particular the character table produced by Clifford-Fischer Theory is in a special format that could not be achieved by direct computations using GAP or Magma. Also providing examples of applications of Clifford-Fischer Theory to both split and non-split extensions is sensible choice, since each group requires individual approach. The readers (particulary young researchers) will highly benefit from the theoretical background required for these computations. GAP and Magma are computational tools and would not replace good powerful and theoretical arguments.

For the notation used in this paper and the description of Clifford-Fischer theory technique, we follow [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14] and [15].

As a subgroup of $GL(6,2)$, the following two elements $\overline{g}_1$ and $\overline{g}_2$ generate the full extension $\overline{G} = 2^{4+5}{:}GL(4,2)$.

$$\overline{g}_1 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \end{pmatrix}, \quad \overline{g}_2 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 \end{pmatrix},$$

with $o(\overline{g}_1) = 30,\ o(\overline{g}_2) = 8$ and $o(\overline{g}_1\overline{g}_2) = 6$.

Now having the group $\overline{G}$ constructed in GAP, it is easy to obtain all its normal subgroups. In fact $\overline{G}$ has five non-trivial proper normal subgroups of orders 16, 32, 256, 512 and 5160960. Let N be the normal subgroup of order 512. One can check that $Z(N) = \Phi(N) = N^{'} \cong 2^4$, the elementary abelian group of order 16, where $Z(N)$, $\Phi(N)$ and $N^{'}$ are the center, Frattini and derived subgroups of N respectively. Further the quotient $N/Z(N)$ is an elementary abelian group of order 32. Thus N is a special 2-group of order 512 of the form 2^{4+5}. Using GAP, it will also be easy to see that $Z(N)$ has no complement in N and thus N can also be written in the form $N = 2^{4}{\cdot}2^{5}$ (non-split). Generators $n_1, n_2, \cdots, n_5$ of N, in terms of 6-dimensional matrices over $\mathbb{F}_2$ can easily be obtained. Also N has 271 involutions and 240 elements of order 4. The elements of N are distributed into 152 conjugacy classes as follows:

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- singleton conjugacy class consisting of the identity element,
- 15 singleton classes, each of which consists of a central involution,
- one conjugacy class consists of 16 non-central involutions,
- 120 conjugacy classes, each of which consists of two non-central involutions,
- 15 conjugacy classes, each of which consists only of 16 elements of order 4.

In Magma or GAP one can check for the complements of N in $\overline{G} = \langle \overline{g}_1, \overline{g}_2 \rangle$, where here we obtained only one complement $G \cong GL(4,2)$. The following two elements g_1 and g_2 generate the complement G of N in $\overline{G}$. Note that G is a subgroup of $\overline{G}$ isomorphic to the quotient $\overline{G}/N \cong GL(4,2)$ and together with N forms the split extension $\overline{G}$ in consideration.

$$g_1 = \begin{pmatrix} 1&0&0&0&0&0\\ 1&0&1&1&0&1\\ 0&0&1&0&0&0\\ 0&0&0&1&0&0\\ 1&1&1&1&1&1\\ 0&0&0&1&1&0 \end{pmatrix}, \quad g_2 = \begin{pmatrix} 1&0&0&0&0&0\\ 1&1&1&0&1&0\\ 0&0&1&0&0&0\\ 0&0&0&1&0&1\\ 0&0&0&0&1&0\\ 0&0&0&0&0&1 \end{pmatrix},$$

with $o(g_1) = 4,\ o(g_2) = 2$ and $o(g_1 g_2) = 15$.

2. Conjugacy classes of $\overline{G} = 2^{4+5}{:}GL(4,2)$

In this section we use the method of the coset analysis technique (see Basheer [1], Basheer and Moori [5, 6] and [8] or Moori [21] and [22] for more details) as we are interested to organize the classes of $\overline{G}$ corresponding to the classes of G.

Example 2.1. Consider the identity coset $N1_G = N$. The action of N on $N1_G = N$ produces 152 orbits and these orbits are the conjugacy classes of N, which were mentioned above, and thus $k_1 = 152$. Now the action of $C_G(1_G)$ on these 152 classes leaves invariant

- the identity class (which will form the identity class of $\overline{G}$),
- the class consisting of the 16 non-central involutions (which will form an involutory class in $\overline{G}$ of size 16),

while it fuses

- the 15 singleton classes (each of which consists of a central involution) into one orbit (forming an involutory class of size 15 in $\overline{G}$),
- out of the 120 classes (each of which consists of two non-central involutions), 15 classes fuse together to form a new orbit (forming an involutory class of size $15 \times 2 = 30$ in $\overline{G}$), and the remaining 105 classes fuse together to form another orbit (and thus an involutory class of size $105 \times 2 = 210$ in $\overline{G}$),

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- the 15 conjugacy classes (each of which consists only of 16 elements of order 4) into a single orbit (and thus forming a class of elements of order 4 of size $15 \times 16 = 240$ in $\overline{G}$).

It follows that in $\overline{G}$, we get 6 conjugacy classes (of sizes 1, 15, 16, 30, 210 and 240 respectively) corresponding to the identity coset.

$g_1 = 1A$	$k_1 = 152$	$m_{11} = 1$	g_{11}	1	1	10321920
		$m_{12} = 15$	g_{12}	2	15	688128
		$m_{13} = 16$	g_{13}	2	16	645120
		$m_{14} = 30$	g_{14}	2	30	344064
		$m_{15} = 210$	g_{15}	2	210	49152
		$m_{16} = 240$	g_{16}	4	240	43008
$g_2 = 2A$	$k_2 = 48$	$m_{21} = 4$	g_{21}	2	420	24576
		$m_{22} = 12$	g_{22}	2	1260	8192
		$m_{23} = 16$	g_{23}	2	1680	6144
		$m_{24} = 24$	g_{24}	2	2520	4096
		$m_{25} = 24$	g_{25}	2	2520	4096
		$m_{26} = 16$	g_{26}	4	1680	6144
		$m_{27} = 16$	g_{27}	4	1680	6144
		$m_{28} = 32$	g_{28}	4	3360	3072
		$m_{29} = 48$	g_{29}	4	5040	2048
		$m_{2,10} = 96$	$g_{2,10}$	4	10080	1024
		$m_{2,11} = 96$	$g_{2,11}$	4	10080	1024
		$m_{2,12} = 128$	$g_{2,12}$	4	13440	768
$g_3 = 2B$	$k_3 = 20$	$m_{31} = 16$	g_{31}	2	3360	3072
		$m_{32} = 16$	g_{32}	2	3360	3072
		$m_{33} = 48$	g_{33}	4	10080	1024
		$m_{34} = 48$	g_{34}	4	10080	1024
		$m_{35} = 96$	g_{35}	4	20160	512
		$m_{36} = 96$	g_{36}	4	20160	512
		$m_{37} = 192$	g_{37}	4	40320	256
$g_4 = 3A$	$k_4 = 2$	$m_{41} = 256$	g_{41}	3	28672	360
		$m_{42} = 256$	g_{42}	6	28672	360

TABLE 2. The conjugacy classes of $\overline{G} = 2^{4+5}{:}(GL(4,2)$.

Next we list the conjugacy classes of $\overline{G}$ in Table 2, where in this table:

- g_i is the i^{th} conjugacy class of G as listed in the Atlas [17]).
- g_{ij} is a representative of a conjugacy class of $\overline{G}$ correspond to $[g_i]_G$.
- k_i is the number of orbits $Q_{i1}, Q_{i2}, \cdots, Q_{ik_i}$ on the action of N on the coset $N\overline{g}_i = Ng_i = 2^{4+5}g_i$. In particular, the action of N on the identity coset N produces the conjugacy classes of N. Thus $k_1 = 152$.
- m_{ij} are weights attached to each class of $\overline{G}$ that will be used later in computing the Fischer matrices of $\overline{G}$. These weights are computed through the formula

$$m_{ij} = [N_{\overline{G}}(N\overline{g}_i) : C_{\overline{G}}(g_{ij})] = |N|\frac{|C_G(g_i)|}{|C_{\overline{G}}(g_{ij})|}. \qquad (2.1)$$

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$g_5 = 3B$	$k_5 = 14$	$m_{51} = 16$	g_{51}	3	17920	576
		$m_{52} = 48$	g_{52}	6	53760	192
		$m_{53} = 64$	g_{53}	6	71680	144
		$m_{54} = 96$	g_{54}	6	107520	96
		$m_{55} = 96$	g_{55}	6	107520	96
		$m_{56} = 192$	g_{56}	6	215040	48
$g_6 = 4A$	$k_2 = 16$	$m_{61} = 16$	g_{61}	4	20160	512
		$m_{62} = 16$	g_{62}	4	20160	512
		$m_{63} = 32$	g_{63}	4	40320	256
		$m_{64} = 32$	g_{64}	4	40320	256
		$m_{65} = 32$	g_{65}	4	40320	256
		$m_{66} = 32$	g_{26}	4	40320	256
		$m_{67} = 32$	g_{67}	4	40320	256
		$m_{68} = 32$	g_{68}	4	40320	256
		$m_{69} = 32$	g_{69}	4	40320	256
		$m_{6,10} = 64$	$g_{6,10}$	4	80640	128
		$m_{6,11} = 64$	$g_{6,11}$	4	80640	128
		$m_{6,12} = 128$	$g_{6,12}$	8	161280	64
$g_7 = 4B$	$k_7 = 6$	$m_{71} = 64$	g_{71}	4	161280	64
		$m_{72} = 64$	g_{72}	4	161280	64
		$m_{73} = 64$	g_{73}	8	161280	64
		$m_{74} = 64$	g_{74}	8	161280	64
		$m_{75} = 128$	g_{75}	8	322560	32
		$m_{76} = 128$	g_{76}	8	322560	32
$g_8 = 5A$	$k_8 = 2$	$m_{81} = 256$	g_{81}	5	344064	30
		$m_{82} = 256$	g_{82}	10	344064	30
$g_9 = 6A$	$k_9 = 2$	$m_{91} = 256$	g_{91}	6	430080	24
		$m_{92} = 256$	g_{92}	6	430080	24
$g_{10} = 6B$	$k_{10} = 6$	$m_{10,1} = 64$	$g_{10,1}$	6	215040	48
		$m_{10,2} = 64$	$g_{10,2}$	6	215040	48
		$m_{10,3} = 64$	$g_{10,3}$	12	215040	48
		$m_{10,4} = 64$	$g_{10,4}$	12	215040	48
		$m_{10,5} = 128$	$g_{10,5}$	12	430080	24
		$m_{10,6} = 128$	$g_{10,6}$	12	430080	24
$g_{11} = 7A$	$k_{11} = 5$	$m_{11,1} = 64$	$g_{11,1}$	7	184320	56
		$m_{11,2} = 64$	$g_{11,2}$	14	184320	56
		$m_{11,3} = 128$	$g_{11,3}$	14	368640	28
		$m_{11,4} = 128$	$g_{11,4}$	14	368640	28
		$m_{11,5} = 128$	$g_{11,5}$	28	368640	28
$g_{12} = 7B$	$k_{12} = 5$	$m_{12,1} = 64$	$g_{12,1}$	7	184320	56
		$m_{12,2} = 64$	$g_{12,2}$	14	184320	56
		$m_{12,3} = 128$	$g_{12,3}$	14	368640	28
		$m_{12,4} = 128$	$g_{12,4}$	14	368640	28
		$m_{12,5} = 128$	$g_{12,5}$	28	368640	28
$g_{13} = 15A$	$k_{13} = 2$	$m_{13,1} = 256$	$g_{13,1}$	15	344064	30
		$m_{13,2} = 256$	$g_{13,2}$	30	344064	30
$g_{14} = 15B$	$k_{14} = 2$	$m_{14,1} = 256$	$g_{14,1}$	15	344064	30
		$m_{14,2} = 256$	$g_{14,2}$	30	344064	30

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3. Inertia factor groups of $\overline{G} = 2^{4+5}{:}GL(4,2)$

We have seen in Section 2 that the action of $\overline{G}$ on the classes of N produced six orbits of lengths 1, 15, 16, 30, 210 and 240. By a theorem of Brauer (see for example Theorem 5.1.1 of [1]), it follows that the action of $\overline{G}$ on $\mathrm{Irr}(N)$ will also produce six orbits. We have used GAP to find the lengths of these orbits (the GAP command "Orbits(G,IrrN)" will return the orbits of G on the set of complex irreducible characters of N, denoted here by IrrN). We found that the orbit lengths are 1, 1, 15, 15, 15 and 105. Recall that N has 152 conjugacy classes, thus $|\mathrm{Irr}(N)| = 152$. Since $N/Z(N)$ is an elementary abelian group of order 32, it follows that N has 32 linear characters (these are the lifts of $\mathrm{Irr}(N/Z(N)) \cong 2^5$ to the full extension N). Also since N is a 2-group, it follows that the degrees of all the other remaining 120 characters are powers of 2. In fact all the other 120 characters have degrees 2 (one can see that $\sum_{i=1}^{|\mathrm{Irr}(N)|} (\chi_i(1_N))^2 = 32 \times 1^2 + 120 \times 2^2 = 512 = |N|$). We may assume that the first four orbits (of lengths 1, 1, 15 and 15) contains the 32 linear characters while the fifth and the sixth orbits (of lengths 15 and 105 respectively) contain the other 120 characters each of which is of degree 2. Now by checking the maximal subgroups of $GL(4,2)$ (see the Atlas), we infer that the first five inertia factor groups are $H_1 = H_2 = GL(4,2)$ and $H_3 = H_4 = H_5 = 2^3{:}GL(3,2)$. Since the sixth inertia factor group H_6 has index 105 in $GL(4,2)$, it follows that it is either an index 7 subgroup in $2^3{:}GL(3,2)$ or an index 3 subgroup of $2^4{:}(S_3 \times S_3)$. The group $2^3{:}GL(3,2)$ has two conjugacy classes of maximal subgroups of order 192. These two class have representatives of the form $2^3{:}S_4$ and $2^4{:}D_{12}$ respectively. On the other hand, if $H_6 \leq 2^4{:}(S_3 \times S_3)$ with index 3, then H_6 must be a split extension of 2^4 by a maximal subgroup (of index 3) of $S_3 \times S_3$. The maximal subgroups of $S_3 \times S_3$ are $3 \times S_3$ (two non-conjugate copies), $(3 \times 3){:}2$ and D_{12} (two non-conjugate copies). It follows that if $H_6 \leq 2^4{:}(S_3 \times S_3)$ with index 3, then H_6 must have the form $2^4{:}D_{12}$. Therefore $H_6 \in \{2^3{:}S_4, 2^4{:}D_{12}\}$. Thus the next step is to determine the structure of H_6 and then for each inertia factor group, we have to determine which projective character table we should use to construct the character table of $\overline{G}$.

Now assume that the first orbit on the action of $\overline{G}$ on $\mathrm{Irr}(N)$ consists of the identity character $\mathbf{1}_N$, which we know that it is extendable to a character of $\overline{G}$ (this is since $\mathbf{1}_{\overline{G}}\downarrow_N^{\overline{G}} = \mathbf{1}_N$). Also since $\overline{G}$ splits over N and the characters of the second, third and fourth orbits are linear, it follows by application of Theorem 5.1.8 of Basheer [1] that these characters are extendable to ordinary characters of their respective inertia groups. Thus for the construction of the character table of $\overline{G}$, all the character tables of the inertia factors H_1, H_2, H_3

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and H_4 that we will use are the ordinary ones. At this stage we are not yet sure whether the characters of the fifth and sixth orbits are extendable to ordinary characters of $\overline{H}_5$ and $\overline{H}_6$. Now since $|\mathrm{Irr}(H_1)| = |\mathrm{Irr}(H_2)| = 14$ and $|\mathrm{Irr}(H_3)| = |\mathrm{Irr}(H_4)| = 11$, we must have

$$|\mathrm{IrrProj}(H_5,\alpha^{-1})|+|\mathrm{IrrProj}(H_6,\beta^{-1})| = |\mathrm{Irr}(\overline{G})|-\sum_{i=1}^{4}|\mathrm{Irr}(H_i)| = 75-50 = 25, \tag{3.1}$$

where α and β are factor sets of the Schur multipliers $M(H_5)$ and $M(H_6)$ respectively.

Remark 3.1. Using GAP we have $|\mathrm{Irr}(H_5)| = 11$, while for H_6, we have $|\mathrm{Irr}(2^3{:}S_4)| = 13$ and $|\mathrm{Irr}(2^4{:}D_{12})| = 14$.

In [23] it was shown that the full representation group of H_5 is of the form $2^2{\cdot}H_5$, with Schur multiplier $M(H_5) \cong 2^2$. Hence H_5 has 4 sets of irreducible projective character tables with associated factor sets α_i^{-1}, $i = 1, 2, 3, 4$, such that $\alpha_i^2 \sim [1]$ for $i = 2, 3, 4$. The set $\mathrm{IrrProj}(H_5, \alpha_1^{-1}) = \mathrm{Irr}(H_5)$. The number of projective characters of H_5 for each of the non-trivial factor sets α_i^{-1}, $i = 2, 3, 4$, are found to be $|\mathrm{IrrProj}(H_5, \alpha_2^{-1})| = 5$, $|\mathrm{IrrProj}(H_5, \alpha_3^{-1})| = 5$ and $|\mathrm{IrrProj}(H_5, \alpha_4^{-1})| = 8$ (see [23]). Now, if $|\mathrm{IrrProj}(H_5, \alpha^{-1})| = 5$ or 8, then it follows from Equation (3.1) that $|\mathrm{IrrProj}(H_6, \beta^{-1})| = 20$ or 17. But it is impossible, since $|\mathrm{IrrProj}(H_6, \beta^{-1})|$ cannot exceed $|\mathrm{Irr}(H_6)|$, which is either 13 or 14 by Remark 3.1. It is a known fact that for any finite group G, the number $|\mathrm{IrrProj}(G, \alpha)|$ of irreducible projective characters of G with associated factor set α cannot exceed the number $|\mathrm{Irr(G)}|$ of ordinary irreducible characters of G (or equivalently stated, that the number of α-regular classes is always less then or equal to the number of conjugacy classes of G). So, the only option left is that $|\mathrm{IrrProj}(H_5, \alpha^{-1})| = |\mathrm{Irr}(H_5)| = 11$, and hence we will use the ordinary irreducible characters of H_5 in the construction of $\mathrm{Irr}(\overline{G})$. From the preceding argument and Equation (3.1) it follows that $|\mathrm{IrrProj}(H_6, \beta^{-1})| = 14$. Since we have $|\mathrm{IrrProj}(H_6, \beta^{-1})| = 14$ and that for each set of irreducible projective characters $\mathrm{IrrProj}(2^3{:}S_4, \alpha)$ of $2^3{:}S_4$ with associated factor set α, $|\mathrm{IrrProj}(2^3{:}S_4, \alpha)| \leq |\mathrm{Irr}(2^3{:}\mathrm{S}_4)| = 11$, it follows that the structure of H_6 is of the form $2^4{:}D_{12}$.

Finally, we have to determine whether we will use the set $\mathrm{Irr}(2^4{:}D_{12})$ or an appropriate set $\mathrm{IrrProj}(2^4{:}D_{12}, \alpha^{-1})$ with non-trivial factor set α^{-1} such that $|\mathrm{IrrProj}(2^4{:}D_{12}, \alpha^{-1})| = 14$. We construct $2^4{:}D_{12}$ as a permutation group within A_8 (see for generators below) using GAP and then compute the Schur Multiplier $M(H_6) = 2^3$ of $H_6 = 2^4{:}D_{12}$. Hence 8 sets of projective character tables with associated factor sets β_i^{-1}, $i = 1, 2, 3, \cdots, 8$ exist for H_6, where $\mathrm{IrrProj}(H_6, \beta_1^{-1}) = \mathrm{Irr}(H_6)$ and $\beta^2 \sim [1]$ for the nontrivial factor sets. The following GAP code (see [24], [25]) is used to find the number of irreducible

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projective characters of H_6 associated with each factor set β_i^{-1}.

```
gap> h := H6;;
gap> f := EpimorphismSchurCover(h);;
gap> f := InverseGeneralMapping(IsomorphismPcGroup(Source(f)))*f;;
gap> z := Kernel(f);; Schur Multiplier M(H6)
gap> x := Source(f);; Representation group for H5
gap> List( Irr(z), lambda-> Number( Irr( x ), chi -> not IsZero(ScalarProduct(
RestrictedClassFunction( chi, z ), lambda ) ) ));
```

The above GAP code is based on the fact that all irreducible projective representations of a finite group G for any factor set α can be "linearized" to an ordinary representation of a full representation group $R = M(G).G$ of G (see [19]). Here $M(G)$ denotes the Schur Multiplier which is isomorphic to the second cohomology group $H^2(G, \mathbb{C}^*)$ and which is abelian. Hence we can obtained all the sets $\mathrm{IrrProj}(G, \alpha)$ of G for any factor set α from $\mathrm{Irr}(R)$. Furthermore, if $\theta \in \mathrm{Irr}M(G)$ is any linear character of $M(G)$, then $\sum_{\chi\in\mathrm{Irr}(R)} \frac{<\chi\downarrow_{M(G)},\theta>}{\chi(1)}$ (see line 6 of above GAP code) is the number of irreducible characters $\mathrm{Irr}(R)$ of R which lie over θ. Now $\sum_{\chi\in\mathrm{Irr}(R)} \frac{<\chi\downarrow_{M(G)},\theta>}{\chi(1)} = |\mathrm{IrrProj}(G,\alpha)|$ for some factor set α. And so we can compute $\sum_{\chi\in\mathrm{Irr}(R)} \frac{<\chi\downarrow_{M(G)},\theta>}{\chi(1)}$ for each linear character $\theta \in \mathrm{Irr}M(G)$ and hence find the number $|\mathrm{IrrProj}(G,\alpha)|$ of irreducible projective characters of G for any factor set α. The above GAP code is very efficient if the group is of relatively small size and is also solvable as in the case of H_6. From the output of the above GAP code, we obtained that the number of irreducible projective characters of H_6 for each associated factor set are 14, 8, 7, 7, 6, 3, 6 and 3. The set containing the 14 irreducible projective characters is associated with the trivial factor set and hence are the ordinary irreducible characters of H_6. Clearly, we can see from the output of the GAP code that $|\mathrm{IrrProj}(H_6, \beta^{-1})|$ for any non-trivial factor set is strictly less than 14. Hence we have the following proposition.

Proposition 3.2. *The sixth inertia factor group H_6 is $2^4{:}D_{12}$ and there is no involvement of projective characters in the construction of the character table of $\overline{G}$.*

Corollary 3.3. *There exists characters $\Theta_5, \Theta_6 \in \mathrm{Irr}(\overline{G})$ with $\deg(\Theta_5) = \deg(\Theta_6) = 2$ such that $\Theta_5\downarrow_N^{\overline{G}} = \theta_5$ and $\Theta_6\downarrow_N^{\overline{G}} = \theta_6$.*

As subgroups of $G \cong A_8$, the inertia factors $H_1 = H_2 = GL(4,2) \cong A_8$, $H_3 = H_4 = H_5$ and H_6 are generated as follows:

$$\begin{aligned} H_1 &= \langle (1,2,3,4,5,6,7), (6,7,8) \rangle, \\ H_3 &= \langle (2,3,5)(4,7,6), (5,7)(6,8), (1,2)(3,4)(5,6)(7,8) \rangle, \\ H_6 &= \langle (3,4)(5,8,6,7), (1,5)(2,8,3,6,4,7) \rangle. \end{aligned}$$

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In Tables 3 and 3 we show the character tables of $H_3 = H_4 = H_5 = 2^3{:}GL(3,2)$ and $H_6 = 2^4{:}D_{12}$ respectively, while in Table 3 we list the fusions of the classes of $H_3 = H_4 = H_5$ and H_6 into the classes of G.

	$1a$	$2a$	$2b$	$2c$	$3a$	$4a$	$4b$	$4c$	$6a$	$7a$	$7b$
$\lvert C_{H_3}(h)\rvert$	1344	192	32	32	6	16	8	8	6	7	7
χ_1	1	1	1	1	1	1	1	1	1	1	1
χ_2	3	3	−1	−1	0	−1	1	1	0	A	$\overline{A}$
χ_3	3	3	−1	−1	0	−1	1	1	0	$\overline{A}$	A
χ_4	6	6	2	2	0	2	0	0	0	−1	−1
χ_5	7	−1	3	−1	1	−1	1	−1	−1	0	0
χ_6	7	7	−1	−1	1	−1	−1	−1	1	0	0
χ_7	7	−1	−1	3	1	−1	−1	1	−1	0	0
χ_8	8	8	0	0	−1	0	0	0	−1	1	1
χ_9	14	−2	2	2	−1	−2	0	0	1	0	0
χ_{10}	21	−3	1	−3	0	1	−1	1	0	0	0
χ_{11}	21	−3	−3	1	0	1	1	−1	0	0	0

TABLE 3. The character table of $H_3 = H_4 = H_5 = 2^3{:}GL(3,2)$.

where in Table 3, $A = -1 - b7 = -\frac{1}{2} - i\frac{\sqrt{7}}{2}$.

	$1a$	$2a$	$2b$	$2c$	$2d$	$2e$	$2f$	$3a$	$4a$	$4b$	$4c$	$4d$	$4e$	$6a$
$\lvert C_{H_6}(h)\rvert$	192	64	48	32	16	32	16	6	16	16	16	8	8	6
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	−1	1	−1	1	1	1	−1	−1	1	−1	1	−1
χ_3	1	1	1	1	−1	1	−1	1	−1	1	−1	−1	−1	1
χ_4	1	1	−1	1	1	1	−1	1	1	−1	−1	1	−1	−1
χ_5	2	2	−2	2	0	2	0	−1	0	−2	0	0	0	1
χ_6	2	2	2	2	0	2	0	−1	0	2	0	0	0	−1
χ_7	3	3	−3	−1	−1	−1	1	0	−1	1	1	1	−1	0
χ_8	3	3	3	−1	−1	−1	−1	0	−1	−1	−1	1	1	0
χ_9	3	3	−3	−1	1	−1	−1	0	1	1	−1	−1	1	0
χ_{10}	3	3	3	−1	1	−1	1	0	1	−1	1	−1	−1	0
χ_{11}	6	−2	0	2	−2	−2	0	0	2	0	0	0	0	0
χ_{12}	6	−2	0	−2	0	2	−2	0	0	0	2	0	0	0
χ_{13}	6	−2	0	−2	0	2	2	0	0	0	−2	0	0	0
χ_{14}	6	−2	0	2	2	−2	0	0	−2	0	0	0	0	0

TABLE 4. The character table of $H_6 = 2^4{:}D_{12}$

.

where in Table 3, $A = -i\sqrt{2}$ and $B = \frac{1}{2} - i\frac{\sqrt{11}}{2}$.

4. Fischer matrices of $\overline{G} = 2^{4+5}{:}GL(4,2)$

Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a finite group extension. For each conjugacy class $[g_i]_G$, there corresponds a unique (up to the permutation of rows and columns) matrix $\mathcal{F}_i$. We refer to this matrix as the ***Fischer matrix*** or ***Clifford matrix*** and it is a very important component for the computations

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Class of H_3	$\hookrightarrow$	Class of $GL(4,2)$	Class of H_6	$\hookrightarrow$	Class of $GL(4,2)$
$1a$		$1A$	$1a$		$1A$
$2a$		$2A$	$2a$		$2A$
$2b$		$2B$	$2b$		$2A$
$2c$		$2A$	$2c$		$2A$
$3a$		$3B$	$2d$		$2A$
$4a$		$4A$	$2e$		$2B$
$4b$		$4B$	$2f$		$2B$
$4c$		$4A$	$3a$		$3B$
$6a$		$6B$	$4a$		$4A$
$7a$		$7A$	$4b$		$4A$
$7b$		$7B$	$4c$		$4A$
			$4d$		$4A$
			$4e$		$4B$
			$6a$		$6B$

TABLE 5. The fusions of $H_3 = H_4 = H_5$ and H_6 into G.

of the character table of $\overline{G}$. For the construction of a Fischer matrix $\mathcal{F}_i$, we refer to [1]. Fischer matrices satisfy some interesting properties and certain orthogonality relations, which help in the computations of their entries. We gather these properties in the following Proposition.

Proposition 4.1. (i) $\sum_{k=1}^{t} c(g_{ik}) = c(g_i)$,

(ii) *$\mathcal{F}_i$ is non-singular for each i,*

(iii) $a_{ij}^{(1,1)} = 1,\ \forall\ 1 \leq j \leq c(g_i)$,

(iv) *If $N\overline{g}_i$ is a split coset, then $a_{i1}^{(k,m)} = \frac{|C_G(g_i)|}{|C_{H_k}(g_{ikm})|},\ \forall i \in \{1, 2, \cdots, r\}$. In particular for the identity coset we have $a_{11}^{(k,m)} = [G : H_k]\theta_k(1_N),\ \forall\ (k,m) \in J_1$,*

(v) *If $N\overline{g}_i$ is a split coset, then $|a_{ij}^{(k,m)}| \leq |a_{i1}^{(k,m)}|$ for all $1 \leq j \leq c(g_i)$. Moreover if $|N| = p^{\alpha}$, for some prime p, then $a_{ij}^{(k,m)} \equiv a_{i1}^{(k,m)} (\mathrm{mod}\ p)$,*

(vi) *For each $1 \leq i \leq r$, the weights m_{ij} satisfy the relation $\sum_{j=1}^{c(g_i)} m_{ij} = |N|$,*

(vii) ***Column Orthogonality Relation:***

$$\sum_{(k,m)\in J_i} |C_{H_k}(g_{ikm})| a_{ij}^{(k,m)} \overline{a_{ij'}^{(k,m)}} = \delta_{jj'} |C_{\overline{G}}(g_{ij})|,$$

(viii) ***Row Orthogonality Relation:***

$$\sum_{j=1}^{c(g_i)} m_{ij} a_{ij}^{(k,m)} \overline{a_{ij}^{(k',m')}} = \delta_{(k,m)(k',m')} a_{i1}^{(k,m)} |N|.$$

We recall from [1, 5] that we label the top and bottom of the columns of the Fischer matrix $\mathcal{F}_i$, corresponding to g_i, by the sizes of the centralizers

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of g_{ij}, $1 \leq j \leq c(g_i)$ in $\overline{G}$ and m_{ij} respectively. In Table 2 we supplied $|C_{\overline{G}}(g_{ij})|$ and m_{ij}, $1 \leq i \leq 14$, $1 \leq j \leq c(g_i)$. Also the fusions of the classes of $H_3 = H_4 = H_5$ and H_6 into classes of G were given in Table 3. Since the size of the Fischer matrix $\mathcal{F}_i$ is $c(g_i)$, it follows from Table 2 that the sizes of the Fischer matrices of $\overline{G}$ range between 2 and 12 for every $i \in \{1, 2, \cdots, 14\}$.

We have used the arithmetical properties of the Fischer matrices, given in Proposition 4.1, to calculate some of the entries of these matrices and also to build an algebraic system of equations. With the help of the symbolic mathematical package Maxima [20], we were able to solve these systems of equations and hence we have computed all the Fischer matrices of $\overline{G}$, which are listed below.

$\mathcal{F}_1$

g_1		g_{11}	g_{12}	g_{13}	g_{14}	g_{15}	g_{16}
$o(g_{1j})$		1	2	2	2	2	4
$\lvert C_{\overline{G}}(g_{1j})\rvert$		10321920	688128	645120	344064	49152	43008
(k,m)	$\lvert C_{H_k}(g_{1km})\rvert$						
$(1,1)$	20160	1	1	1	1	1	1
$(2,1)$	20160	1	1	-1	1	1	-1
$(3,1)$	1344	15	15	15	-1	-1	-1
$(4,1)$	1344	15	15	-15	-1	-1	1
$(5,1)$	1344	30	-2	0	14	-2	0
$(6,1)$	192	210	-14	0	-14	2	0
m_{1j}		1	1	15	15	15	105

$\mathcal{F}_2$

g_2		g_{21}	g_{22}	g_{23}	g_{24}	g_{25}	g_{26}	g_{27}	g_{28}	g_{29}	$g_{2,10}$	$g_{2,11}$	$g_{2,12}$
$o(g_{2j})$		2	2	2	2	2	4	4	4	4	4	4	4
$\lvert C_{\overline{G}}(g_{2j})\rvert$		24576	8192	6144	4096	4096	6144	6144	3072	2048	1024	1024	768
(k,m)	$\lvert C_{H_k}(g_{2km})\rvert$												
$(1,1)$	192	1	1	1	1	1	1	1	1	1	1	1	1
$(2,1)$	192	1	1	-1	1	1	1	-1	1	1	1	-1	-1
$(3,1)$	192	1	1	1	1	1	1	1	-1	1	-1	1	-1
$(3,2)$	32	6	6	6	-2	-2	6	6	0	-2	0	-2	0
$(4,1)$	192	1	1	-1	1	1	1	-1	-1	1	-1	-1	1
$(4,2)$	32	6	6	-6	-2	-2	6	-6	0	-2	0	2	0
$(5,1)$	192	2	2	0	2	2	-2	0	0	-2	0	0	0
$(5,2)$	32	12	-4	0	-4	4	0	0	6	0	-2	0	0
$(6,1)$	64	6	6	0	-2	-2	-6	0	0	2	0	0	0
$(6,2)$	48	0	0	8	0	0	0	-8	0	0	0	0	0
$(6,3)$	32	12	-4	0	-4	4	0	0	-6	0	2	0	0
$(6,4)$	16	24	-8	0	8	-8	0	0	0	0	0	0	0
m_{2j}		4	12	16	24	24	16	16	32	48	96	96	128

$\mathcal{F}_3$

g_3		g_{31}	g_{32}	g_{33}	g_{34}	g_{35}	g_{36}	g_{37}
$o(g_{3j})$		2	2	4	4	4	4	4
$\lvert C_{\overline{G}}(g_{3j})\rvert$		3072	3072	1024	1024	512	512	256
(k,m)	$\lvert C_{H_k}(g_{3km})\rvert$							
$(1,1)$	96	1	1	1	1	1	1	1
$(2,1)$	96	1	-1	-1	1	1	1	-1
$(3,1)$	32	3	3	3	3	-1	-1	-1
$(4,1)$	32	3	-3	-3	3	-1	-1	1
$(5,1)$	32	6	0	0	-2	2	-2	0
$(6,1)$	32	6	0	0	-2	-2	2	0
$(6,2)$	16	0	12	-4	0	0	0	0
m_{3j}		16	16	48	48	96	96	192

$\mathcal{F}_4$

g_4		g_{41}	g_{42}
$o(g_{4j})$		3	6
$\lvert C_{\overline{G}}(g_{4j})\rvert$		360	360
(k,m)	$\lvert C_{H_k}(g_{4km})\rvert$		
$(1,1)$	180	1	1
$(2,1)$	180	1	-1
m_{4j}		256	256

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$\mathcal{F}_5$

g_5		g_{51}	g_{52}	g_{53}	g_{54}	g_{55}	g_{56}
$o(g_{5j})$		3	6	6	6	6	12
$\lvert C_{\overline{G}}(g_{5j})\rvert$		576	192	144	96	96	48
(k,m)	$\lvert C_{H_k}(g_{5km})\rvert$						
$(1,1)$	18	1	1	1	1	1	1
$(2,1)$	18	1	1	−1	1	1	−1
$(3,1)$	6	3	3	3	−1	−1	−1
$(4,1)$	6	3	3	−3	−1	−1	1
$(5,1)$	6	6	−2	0	2	−2	0
$(6,1)$	6	6	−2	0	−2	2	0
m_{5j}		16	48	64	96	96	192

$\mathcal{F}_6$

g_6		g_{61}	g_{62}	g_{63}	g_{64}	g_{65}	g_{66}	g_{67}	g_{68}	g_{69}	$g_{6,10}$	$g_{6,11}$	$g_{6,12}$
$o(g_{6j})$		4	4	4	4	4	4	4	4	4	4	4	8
$\lvert C_{\overline{G}}(g_{6j})\rvert$		512	512	256	256	256	256	256	256	256	128	128	64
(k,m)	$\lvert C_{H_k}(g_{6km})\rvert$												
$(1,1)$	16	1	1	1	1	1	1	1	1	1	1	1	1
$(2,1)$	16	1	1	1	1	1	−1	−1	−1	−1	1	1	−1
$(3,1)$	16	1	1	1	1	1	1	1	1	1	−1	−1	−1
$(3,2)$	8	2	2	−2	2	−2	−2	2	−2	2	0	0	0
$(4,1)$	16	1	1	1	1	1	−1	−1	−1	−1	−1	−1	1
$(4,2)$	8	2	2	−2	2	−2	2	−2	2	−2	0	0	0
$(5,1)$	16	2	2	−2	−2	2	0	0	0	0	0	0	0
$(5,2)$	8	−4	4	0	0	0	0	0	0	0	2	−2	0
$(6,1)$	16	2	2	2	−2	−2	0	0	0	0	0	0	0
$(6,2)$	16	0	0	0	0	0	2	−2	−2	2	0	0	0
$(6,3)$	16	0	0	0	0	0	2	2	−2	−2	0	0	0
$(6,4)$	8	4	−4	0	0	0	0	0	0	0	2	−2	0
m_{6j}		16	16	32	32	32	32	32	32	32	64	64	128

$\mathcal{F}_7$

g_7		g_{71}	g_{72}	g_{73}	g_{74}	g_{75}	g_{76}
$o(g_{7j})$		4	4	8	8	8	8
$\lvert C_{\overline{G}}(g_{7j})\rvert$		64	64	64	64	32	32
(k,m)	$\lvert C_{H_k}(g_{7km})\rvert$						
$(1,1)$	8	1	1	1	1	1	1
$(2,1)$	8	1	−1	−1	1	−1	1
$(3,1)$	8	1	1	1	1	−1	−1
$(4,1)$	8	1	−1	−1	1	1	−1
$(5,1)$	8	2	0	0	−2	0	0
$(6,1)$	8	0	−2	2	0	0	0
m_{7j}		64	64	64	64	128	128

$\mathcal{F}_8$

g_8		g_{81}	g_{82}
$o(g_{8j})$		5	10
$\lvert C_{\overline{G}}(g_{4j})\rvert$		30	30
(k,m)	$\lvert C_{H_k}(g_{8km})\rvert$		
$(1,1)$	15	1	1
$(2,1)$	15	1	−1
m_{8j}		256	256

$\mathcal{F}_9$

g_9		g_{91}	g_{92}
$o(g_{9j})$		6	6
$\lvert C_{\overline{G}}(g_{9j})\rvert$		24	24
(k,m)	$\lvert C_{H_k}(g_{9km})\rvert$		
$(1,1)$	12	1	1
$(2,1)$	12	1	−1
m_{9j}		256	256

$\mathcal{F}_{10}$

g_{10}		$g_{10,1}$	$g_{10,2}$	$g_{10,3}$	$g_{10,4}$	$g_{10,5}$	$g_{10,6}$
$o(g_{10j})$		6	6	12	12	12	12
$\lvert C_{\overline{G}}(g_{10j})\rvert$		48	48	48	48	24	24
(k,m)	$\lvert C_{H_k}(g_{10km})\rvert$						
$(1,1)$	6	1	1	1	1	1	1
$(2,1)$	6	1	−1	−1	1	−1	1
$(3,1)$	6	1	1	1	1	−1	−1
$(4,1)$	6	1	−1	−1	1	1	−1
$(5,1)$	6	2	0	0	−2	0	0
$(6,1)$	6	0	−2	2	0	0	0
m_{10j}		64	64	64	64	128	128

$\mathcal{F}_{11}$

g_{11}		$g_{11,1}$	$g_{11,2}$	$g_{11,3}$	$g_{11,4}$	$g_{11,5}$
$o(g_{11j})$		7	14	14	14	28
$\lvert C_{\overline{G}}(g_{11j})\rvert$		56	56	28	28	28
(k,m)	$\lvert C_{H_k}(g_{11km})\rvert$					
$(1,1)$	7	1	1	1	1	1
$(2,1)$	7	1	1	−1	1	−1
$(3,1)$	7	1	1	1	−1	−1
$(4,1)$	7	1	1	−1	−1	1
$(5,1)$	7	2	−2	0	0	0
m_{11j}		64	64	128	128	128

$\mathcal{F}_{12}$

g_{12}		$g_{12,1}$	$g_{12,2}$	$g_{12,3}$	$g_{12,4}$	$g_{12,5}$
$o(g_{12j})$		7	14	14	14	28
$\lvert C_{\overline{G}}(g_{12j})\rvert$		56	56	28	28	28
(k,m)	$\lvert C_{H_k}(g_{12km})\rvert$					
$(1,1)$	7	1	1	1	1	1
$(2,1)$	7	1	1	−1	1	−1
$(3,1)$	7	1	1	1	−1	−1
$(4,1)$	7	1	1	−1	−1	1
$(5,1)$	7	2	−2	0	0	0
m_{12j}		64	64	128	128	128

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$\mathcal{F}_{13}$			
g_{13}		$g_{13,1}$	$g_{13,2}$
$o(g_{13j})$		15	30
$\|C_{\overline{G}}(g_{13j})\|$		30	30
(k,m)	$\|C_{H_k}(g_{13km})\|$		
$(1,1)$	15	1	1
$(2,1)$	15	1	-1
m_{13j}		256	256

$\mathcal{F}_{14}$			
g_{14}		$g_{14,1}$	$g_{14,2}$
$o(g_{14j})$		15	30
$\|C_{\overline{G}}(g_{14j})\|$		30	30
(k,m)	$\|C_{H_k}(g_{14km})\|$		
$(1,1)$	15	1	1
$(2,1)$	15	1	-1
m_{14j}		256	256

5. The character table of $\overline{G} = 2^{4+5}{:}(GL(4,2)$

Let $\overline{G} = N{\cdot}G$, where $N \lhd \overline{G}$ and $\overline{G}/N \cong G$, be a finite group extension. By the description of [1, 5], it follows that the full character table of $\overline{G}$ can be constructed easily. Let

- $g_1, g_2, \cdots, g_r$ be representatives for the conjugacy classes of $G \cong \overline{G}/N$. For each $i \in \{1, 2, \cdots, r\}$, let $g_{i1},\ g_{i2},\ \cdots,\ g_{ic(g_i)}$ be representatives for the conjugacy classes of $\overline{G}$, correspond to the class $[g_i]_G$, obtained using the coset analysis technique (see [1] for more details),
- $\mathcal{K}_{ik}$ be the fragment of the projective character table of H_k, with factor set α_k^{-1}, consisting of columns correspond to the $\alpha_k^{-1}-$regular classes of H_k that fuse to $[g_i]_G$ (let such classes be represented by $g_{ik1}, g_{ik2}, \cdots, g_{ikc(g_{ik})}$) and
- $\mathcal{F}_{ik}$ be the sub-matrix of the Fischer matrix $\mathcal{F}_i$ with rows correspond to the pairs (k, g_{ik1}), (k, g_{ik2}), $\cdots$, $(k, g_{ikc(g_{ik})})$ or for brevity $(k, 1)$, $(k, 2)$, $\cdots$, $(k, c(g_{ik}))$ as described by Equation (3) of [9].

For each $i \in \{1, 2, \cdots, r\}$ and $k \in \{1, 2, \cdots, t\}$, where t is the number of the inertia factor groups (that is the number of orbits on the action of G on $\mathrm{Irr}(N)$), the part of the character table of $\overline{G}$ on the classes $[g_{ij}]_{\overline{G}}$, $1 \le j \le c(g_i)$, is given by $\mathcal{K}_{ik}\mathcal{F}_{ik}$. Note that the size of $\mathcal{K}_{ik}$ is $|\mathrm{IrrProj}(H_k, \alpha_k^{-1})| \times c(g_{ik})$, while the size of $\mathcal{F}_{ik}$ is $c(g_{ik}) \times c(g_i)$ and thus $\mathcal{K}_{ik}\mathcal{F}_{ik}$ is of size $|\mathrm{IrrProj}(H_k, \alpha_k^{-1})| \times c(g_i)$. If we let $\mathcal{K}_s$, $s \in \{1, 2, \cdots, t\}$, be the irreducible characters of $\overline{G}$ correspond to the inertia factor group H_k, then the character table of $\overline{G}$ in the format of Clifford-Fischer theory will be composed of the $r \times t$ parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$ and will have the form of Table 5 below.

$[g_i]_G$	g_1				g_2				$\cdots$	g_r			
$[g_{ij}]_{\overline{G}}$	g_{11}	g_{12}	$\cdots$	$g_{1c(g_1)}$	g_{21}	g_{22}	$\cdots$	$g_{2c(g_2)}$	$\cdots$	g_{r1}	g_{r2}	$\cdots$	$g_{rc(g_r)}$
$\mathcal{K}_1$	$\mathcal{K}_{11}\mathcal{F}_{11}$				$\mathcal{K}_{12}\mathcal{F}_{12}$				$\cdots$	$\mathcal{K}_{1r}\mathcal{F}_{1r}$			
$\mathcal{K}_2$	$\mathcal{K}_{21}\mathcal{F}_{21}$				$\mathcal{K}_{22}\mathcal{F}_{22}$				$\cdots$	$\mathcal{K}_{2r}\mathcal{F}_{2r}$			
$\vdots$	$\vdots$				$\vdots$				$\ddots$	$\vdots$			
$\mathcal{K}_t$	$\mathcal{K}_{t1}\mathcal{F}_{t1}$				$\mathcal{K}_{t2}\mathcal{F}_{t2}$				$\cdots$	$\mathcal{K}_{tr}\mathcal{F}_{tr}$			

Table 6. The character table of a group extension $\overline{G}$.

Note 5.1. From Note 3.4 of [9] we know that characters of $\overline{G}$ consisted in $\mathcal{K}_1$ are just $\mathrm{Irr}(G)$ and therefore the size of $\mathcal{K}_{1i}\mathcal{F}_{1i}$, for each $1 \le i \le r$, is $|\mathrm{Irr}(G)| \times c(g_i)$.

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In particular, columns of $\mathcal{K}_{11}\mathcal{F}_{11}$ are the degrees of irreducible characters of G repeated themselves $c(g_1)$ times, where we know that $c(g_1)$ is number of $\overline{G}-$conjugacy classes obtained from the normal subgroup N.

From the Atlas, Sections 2, 3 and 4 we have

- the conjugacy classes of $\overline{G}$ (Table 2),
- the character tables of the inertia factors $H_1 = H_2$ (see the Atlas), $H_3 = H_4 = H_5$ and H_6 (Tables 3 and 3),
- the fusions of classes of the inertia factors $H_3 = H_4 = H_5$ and H_6 into classes of G (Table 3),
- the Fischer matrices of $\overline{G}$ (see Section 4).

Applying the above, it follows that the full character table of $\overline{G}$ can be constructed easily. We obtained that character table of $\overline{G}$ is a 75×75 $\mathbb{C}$-valued matrix, partitioned into 84 parts $\mathcal{K}_{ik}\mathcal{F}_{ik}$, where $1 \leq i \leq 14,\ 1 \leq k \leq 6$. In Table 6 we show the full character table of $\overline{G}$, in the format of Clifford-Fischer theory. We would like to remark that the accuracy of this character table has been tested using GAP. We conclude by mentioning that in the character table of $\overline{G}$ we supplied the fusions of the conjugacy classes of elements of $\overline{G}$ into the conjugacy classes of the group $AGL(6,2) = 2^5{:}GL(5,2) = M$. Here the conjugacy classes of M can be identified according to the size of the centralizers of elements, where if nA, nB and nC are some conjugacy classes of elements of order n in M, then nA will stand for the conjugacy class of the smallest size (largest centralizer size) of elements of order n; nB is the second conjugacy class of smallest size of elements of order n and so on. For example, $2A$ and $2B$ stands for the classes of elements in M with the centralizer sizes 10321920 and 344064 respectively. Similarly it is done for all the other classes of M.

6. Conclusion

In this paper we considered the group $\overline{G} = 2^{4+5}{:}GL(4,2)$, which is the largest maximal subgroup of the affine general linear group $2^5{:}GL(5,2)$ of $GL(6,2)$. This is a split extension of the special 2-group 2^{4+5} by $GL(4,2)$. We used the coset analysis technique to construct its conjugacy classes, where correspond to the 14 conjugacy classes of $GL(4,2)$, we obtained 75 conjugacy classes in $\overline{G}$. Then the inertia factor groups were determined using various techniques including building subroutines in GAP. Here we would like to mention that the GAP commands and other techniques introduced in this paper will be of high benefits to other researchers working in this field. Then we determined the Fischer matrices of the extension $\overline{G}$ using the properties mentioned in Proposition 4.1 together with the symbolic package Maxima.

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Table 6 (continued)

$[g_i]_{GL(4,2)}$	$2B$							$3A$		$3B$						$4A$											
$[g_{ij}]_{\overline{G}}$	$2j$	$2k$	$4i$	$4j$	$4k$	$4l$	$4m$	$3a$	$6a$	$3b$	$6b$	$6c$	$6d$	$6e$	$12a$	$4n$	$4o$	$4p$	$4q$	$4r$	$4s$	$4t$	$4u$	$4v$	$4w$	$4x$	$8a$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	3072	2048	1024	1024	512	512	256	360	360	384	192	144	96	96	48	512	512	256	256	256	256	256	256	256	128	128	64
$\hookrightarrow 2^5{:}GL(5,2)$	$2E$	$2D$	$4C$	$4C$	$4F$	$4G$	$4H$	$3B$	$6B$	$3A$	$6A$	$6A$	$6C$	$6D$	$12A$	$4B$	$4D$	$4E$	$4D$	$4E$	$4F$	$4H$	$4G$	$4H$	$4I$	$4J$	$8A$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	3	3	3	3	3	3	3	4	4	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_3	2	2	2	2	2	2	2	−1	−1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
χ_4	4	4	4	4	4	4	4	5	5	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_5	1	1	1	1	1	1	1	6	6	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_6	1	1	1	1	1	1	1	−3	−3	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_7	1	1	1	1	1	1	1	−3	−3	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_8	4	4	4	4	4	4	4	1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_9	−5	−5	−5	−5	−5	−5	−5	5	5	2	2	2	2	2	2	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_{10}	−3	−3	−3	−3	−3	−3	−3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_{11}	−3	−3	−3	−3	−3	−3	−3	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	1	1
χ_{12}	0	0	0	0	0	0	0	−4	−4	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{13}	0	0	0	0	0	0	0	4	4	−2	−2	−2	−2	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	2	2	2	2	2	2	2	−5	−5	1	1	1	1	1	1	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
χ_{15}	−1	1	1	−1	1	1	−1	1	−1	1	1	−1	1	1	−1	1	1	1	−1	−1	1	1	−1	−1	1	1	−1
χ_{16}	−3	3	3	−3	3	3	−3	4	−4	1	1	−1	1	1	−1	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
χ_{17}	−2	2	2	−2	2	2	−2	−1	1	2	2	−2	2	2	−2	2	2	2	−2	−2	2	2	−2	−2	2	2	−2
χ_{18}	−4	4	4	−4	4	4	−4	5	−5	−1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{19}	−1	1	1	−1	1	1	−1	6	−6	0	0	0	0	0	0	1	1	1	−1	−1	1	1	−1	−1	1	1	−1
χ_{20}	−1	1	1	−1	1	1	−1	−3	3	0	0	0	0	0	0	1	1	1	−1	−1	1	1	−1	−1	1	1	−1
χ_{21}	−1	1	1	−1	1	1	−1	−3	3	0	0	0	0	0	0	1	1	1	−1	−1	1	1	−1	−1	1	1	−1
χ_{22}	−4	4	4	−4	4	4	−4	1	−1	1	1	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{23}	5	−5	−5	5	−5	−5	5	5	−5	2	2	−2	2	2	−2	−1	−1	−1	1	1	−1	−1	1	1	−1	−1	1
χ_{24}	3	−3	−3	3	−3	−3	3	0	0	0	0	0	0	0	0	1	1	1	−1	−1	1	1	−1	−1	1	1	−1
χ_{25}	3	−3	−3	3	−3	−3	3	0	0	0	0	0	0	0	0	1	1	1	−1	−1	1	1	−1	−1	1	1	−1
χ_{26}	0	0	0	0	0	0	0	−4	4	−1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{27}	0	0	0	0	0	0	0	4	−4	−2	−2	2	−2	−2	2	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	−2	2	2	−2	2	2	−2	−5	5	1	1	−1	1	1	−1	−2	−2	−2	2	2	−2	−2	2	2	−2	−2	2
χ_{29}	3	3	3	3	−1	−1	−1	0	0	3	3	3	−1	−1	−1	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1
χ_{30}	−3	−3	−3	−3	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	−3	−3	−3	−3	1	1	1
χ_{31}	−3	−3	−3	−3	1	1	1	0	0	0	0	0	0	0	0	1	1	1	1	1	−3	−3	−3	−3	1	1	1
χ_{32}	6	6	6	6	−2	−2	−2	0	0	0	0	0	0	0	0	2	2	2	2	2	2	2	2	2	−2	−2	−2
χ_{33}	−3	−3	−3	−3	1	1	1	0	0	3	3	3	−1	−1	−1	1	1	1	1	1	−3	−3	−3	−3	1	1	1
χ_{34}	−3	−3	−3	−3	1	1	1	0	0	3	3	3	−1	−1	−1	−3	−3	−3	−3	−3	1	1	1	1	1	1	1
χ_{35}	9	9	9	9	−3	−3	−3	0	0	3	3	3	−1	−1	−1	−3	−3	−3	−3	−3	1	1	1	1	1	1	1
χ_{36}	0	0	0	0	0	0	0	0	0	−3	−3	−3	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{37}	6	6	6	6	−2	−2	−2	0	0	−3	−3	−3	1	1	1	−2	−2	−2	−2	−2	−2	−2	−2	−2	2	2	2
χ_{38}	3	3	3	3	−1	−1	−1	0	0	0	0	0	0	0	0	3	3	3	3	3	−1	−1	−1	−1	−1	−1	−1
χ_{39}	−9	−9	−9	−9	3	3	3	0	0	0	0	0	0	0	0	−1	−1	−1	−1	−1	3	3	3	3	−1	−1	−1
χ_{40}	−3	3	3	−3	−1	−1	1	0	0	3	3	−3	−1	−1	1	3	3	3	−3	−3	−1	−1	1	1	−1	−1	1
χ_{41}	3	−3	−3	3	1	1	−1	0	0	0	0	0	0	0	0	1	1	1	−1	−1	−3	−3	3	3	1	1	−1
χ_{42}	3	−3	−3	3	1	1	−1	0	0	0	0	0	0	0	0	1	1	1	−1	−1	−3	−3	3	3	1	1	−1
χ_{43}	−6	6	6	−6	−2	−2	2	0	0	0	0	0	0	0	0	2	2	2	−2	−2	2	2	−2	−2	−2	−2	2
χ_{44}	3	−3	−3	3	1	1	−1	0	0	3	3	−3	−1	−1	1	1	1	1	−1	−1	−3	−3	3	3	1	1	−1
χ_{45}	3	−3	−3	3	1	1	−1	0	0	3	3	−3	−1	−1	1	−3	−3	−3	3	3	1	1	−1	−1	1	1	−1
χ_{46}	−9	9	9	−9	−3	−3	3	0	0	3	3	−3	−1	−1	1	−3	−3	−3	3	3	1	1	−1	−1	1	1	−1
χ_{47}	0	0	0	0	0	0	0	0	0	−3	−3	3	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0
χ_{48}	−6	6	6	−6	−2	−2	2	0	0	−3	−3	3	1	1	−1	−2	−2	−2	2	2	−2	−2	2	2	2	2	−2
χ_{49}	9	−9	−9	9	3	3	−3	0	0	0	0	0	0	0	0	−1	−1	−1	1	1	3	3	−3	−3	−1	−1	1
χ_{50}	−3	3	3	−3	−1	−1	1	0	0	0	0	0	0	0	0	3	3	3	−3	−3	−1	−1	1	1	−1	−1	1
χ_{51}	0	6	−2	0	2	−2	0	0	0	6	−2	0	2	−2	0	6	−2	−2	0	0	2	−2	0	0	2	−2	0
χ_{52}	0	−6	2	0	−2	2	0	0	0	0	0	0	0	0	0	2	−6	2	0	0	−2	2	0	0	2	−2	0
χ_{53}	0	−6	2	0	−2	2	0	0	0	0	0	0	0	0	0	2	−6	2	0	0	−2	2	0	0	2	−2	0
χ_{54}	0	12	−4	0	4	−4	0	0	0	0	0	0	0	0	0	4	4	−4	0	0	4	−4	0	0	0	0	0
χ_{55}	0	−6	2	0	−2	2	0	0	0	6	−2	0	2	−2	0	2	−6	2	0	0	−2	2	0	0	2	−2	0
χ_{56}	0	−6	2	0	−2	2	0	0	0	6	−2	0	2	−2	0	−6	2	2	0	0	−2	2	0	0	−2	2	0
χ_{57}	0	18	−6	0	6	−6	0	0	0	6	−2	0	2	−2	0	−6	2	2	0	0	−2	2	0	0	−2	2	0
χ_{58}	0	0	0	0	0	0	0	0	0	−6	2	0	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{59}	0	12	−4	0	4	−4	0	0	0	−6	2	0	−2	2	0	−4	−4	4	0	0	−4	4	0	0	0	0	0
χ_{60}	0	6	−2	0	2	−2	0	0	0	0	0	0	0	0	0	6	−2	−2	0	0	2	−2	0	0	2	−2	0
χ_{61}	0	−18	6	0	−6	6	0	0	0	0	0	0	0	0	0	−2	6	−2	0	0	2	−2	0	0	−2	2	0
χ_{62}	−12	6	−2	4	−2	2	0	0	0	6	−2	0	−2	2	0	6	−2	−2	−4	4	−2	2	0	0	−2	2	0
χ_{63}	12	6	−2	−4	−2	2	0	0	0	6	−2	0	−2	2	0	6	−2	−2	4	−4	−2	2	0	0	−2	2	0
χ_{64}	12	6	−2	−4	−2	2	0	0	0	6	−2	0	−2	2	0	−6	2	2	0	0	2	−2	−4	4	2	−2	0
χ_{65}	−12	6	−2	4	−2	2	0	0	0	6	−2	0	−2	2	0	−6	2	2	0	0	2	−2	4	−4	2	−2	0
χ_{66}	0	12	−4	0	−4	4	0	0	0	−6	2	0	2	−2	0	0	0	0	−4	4	0	0	−4	4	0	0	0
χ_{67}	0	12	−4	0	−4	4	0	0	0	−6	2	0	2	−2	0	0	0	0	4	−4	0	0	4	−4	0	0	0
χ_{68}	−12	−6	2	4	2	−2	0	0	0	0	0	0	0	0	0	−2	6	−2	0	0	−2	2	4	−4	2	−2	0
χ_{69}	12	−6	2	−4	2	−2	0	0	0	0	0	0	0	0	0	−2	6	−2	0	0	−2	2	−4	4	2	−2	0
χ_{70}	12	−6	2	−4	2	−2	0	0	0	0	0	0	0	0	0	2	−6	2	4	−4	2	−2	0	0	−2	2	0
χ_{71}	−12	−6	2	4	2	−2	0	0	0	0	0	0	0	0	0	2	−6	2	−4	4	2	−2	0	0	−2	2	0
χ_{72}	0	−12	4	0	4	−4	0	0	0	0	0	0	0	0	0	−4	−4	4	0	0	4	−4	0	0	0	0	0
χ_{73}	0	−12	4	0	4	−4	0	0	0	0	0	0	0	0	0	4	4	−4	0	0	−4	4	0	0	0	0	0
χ_{74}	24	12	−4	−8	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	−4	4	0	0	4	−4	0	0	0
χ_{75}	−24	12	−4	8	−4	4	0	0	0	0	0	0	0	0	0	0	0	0	4	−4	0	0	−4	4	0	0	0

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$[g_i]_{GL(4,2)}$	$1A$						$2A$											
$[g_{ij}]_{\overline{G}}$	$1a$	$2a$	$2b$	$2c$	$2d$	$4a$	$2e$	$2f$	$2g$	$2h$	$2i$	$4b$	$4c$	$4d$	$4e$	$4f$	$4g$	$4h$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	10321920	688128	645120	344064	49152	43008	24576	8192	6144	4096	4096	6144	6144	3072	2048	1024	1024	768
$\hookrightarrow 2^5:GL(5,2)$	$1A$	$2A$	$2A$	$2B$	$2C$	$4A$	$2B$	$2C$	$2C$	$2D$	$2E$	$4A$	$4A$	$4B$	$4C$	$4C$	$4D$	$4E$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	7	7	7	7	7	7	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1
χ_3	14	14	14	14	14	14	6	6	6	6	6	6	6	6	6	6	6	6
χ_4	20	20	20	20	20	20	4	4	4	4	4	4	4	4	4	4	4	4
χ_5	21	21	21	21	21	21	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_6	21	21	21	21	21	21	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_7	21	21	21	21	21	21	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_8	28	28	28	28	28	28	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4	−4
χ_9	35	35	35	35	35	35	3	3	3	3	3	3	3	3	3	3	3	3
χ_{10}	45	45	45	45	45	45	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{11}	45	45	45	45	45	45	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3	−3
χ_{12}	56	56	56	56	56	56	8	8	8	8	8	8	8	8	8	8	8	8
χ_{13}	64	64	64	64	64	64	0	0	0	0	0	0	0	0	0	0	0	0
χ_{14}	70	70	70	70	70	70	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2	−2
χ_{15}	1	1	−1	1	1	−1	1	1	−1	1	1	1	−1	1	1	−1	1	−1
χ_{16}	7	7	−7	7	7	−7	−1	−1	1	−1	−1	−1	1	−1	−1	1	−1	1
χ_{17}	14	14	−14	14	14	−14	6	6	−6	6	6	6	−6	6	6	−6	6	−6
χ_{18}	20	20	−20	20	20	−20	4	4	−4	4	4	4	−4	4	4	−4	4	−4
χ_{19}	21	21	−21	21	21	−21	−3	−3	3	−3	−3	−3	3	−3	−3	3	−3	3
χ_{20}	21	21	−21	21	21	−21	−3	−3	3	−3	−3	−3	3	−3	−3	3	−3	3
χ_{21}	21	21	−21	21	21	−21	−3	−3	3	−3	−3	−3	3	−3	−3	3	−3	3
χ_{22}	28	28	−28	28	28	−28	−4	−4	4	−4	−4	−4	4	−4	−4	4	−4	4
χ_{23}	35	35	−35	35	35	−35	3	3	−3	3	3	3	−3	3	3	−3	3	−3
χ_{24}	45	45	−45	45	45	−45	−3	−3	3	−3	−3	−3	3	−3	−3	3	−3	3
χ_{25}	45	45	−45	45	45	−45	−3	−3	3	−3	−3	−3	3	−3	−3	3	−3	3
χ_{26}	56	56	−56	56	56	−56	8	8	−8	8	8	8	−8	8	8	−8	8	−8
χ_{27}	64	64	−64	64	64	−64	0	0	0	0	0	0	0	0	0	0	0	0
χ_{28}	70	70	−70	70	70	−70	−2	−2	2	−2	−2	−2	2	−2	−2	2	−2	2
χ_{29}	15	15	15	−1	−1	−1	7	7	7	−1	−1	7	7	−1	−1	−1	−1	−1
χ_{30}	45	45	45	−3	−3	−3	−3	−3	−3	5	5	−3	−3	−3	5	5	−3	−3
χ_{31}	45	45	45	−3	−3	−3	−3	−3	−3	5	5	−3	−3	−3	5	5	−3	−3
χ_{32}	90	90	90	−6	−6	−6	18	18	18	2	2	18	18	−6	2	2	−6	−6
χ_{33}	105	105	105	−7	−7	−7	17	17	17	−7	−7	17	17	1	−7	−7	1	1
χ_{34}	105	105	105	−7	−7	−7	1	1	1	9	9	1	1	−7	9	9	−7	−7
χ_{35}	105	105	105	−7	−7	−7	−7	−7	−7	1	1	−7	−7	1	1	1	1	1
χ_{36}	120	120	120	−8	−8	−8	8	8	8	8	8	8	8	−8	8	8	−8	−8
χ_{37}	210	210	210	−14	−14	−14	10	10	10	−6	−6	10	10	2	−6	−6	2	2
χ_{38}	315	315	315	−21	−21	−21	−21	−21	−21	3	3	−21	−21	3	3	3	3	3
χ_{39}	315	315	315	−21	−21	−21	3	3	3	−5	−5	3	3	3	−5	−5	3	3
χ_{40}	15	15	−15	−1	−1	1	7	7	−7	−1	−1	7	−7	−1	−1	1	−1	1
χ_{41}	45	45	−45	−3	−3	3	−3	−3	3	5	5	−3	3	−3	5	−5	−3	3
χ_{42}	45	45	−45	−3	−3	3	−3	−3	3	5	5	−3	3	−3	5	−5	−3	3
χ_{43}	90	90	−90	−6	−6	6	18	18	−18	2	2	18	−18	−6	2	−2	−6	6
χ_{44}	105	105	−105	−7	−7	7	17	17	−17	−7	−7	17	−17	1	−7	7	1	−1
χ_{45}	105	105	−105	−7	−7	7	1	1	−1	9	9	1	−1	−7	9	−9	−7	7
χ_{46}	105	105	−105	−7	−7	7	−7	−7	7	1	1	−7	7	1	1	−1	1	−1
χ_{47}	120	120	−120	−8	−8	8	8	8	−8	8	8	8	−8	−8	8	−8	−8	8
χ_{48}	210	210	−210	−14	−14	14	10	10	−10	−6	−6	10	−10	2	−6	6	2	−2
χ_{49}	315	315	−315	−21	−21	21	3	3	−3	−5	−5	3	−3	3	−5	5	3	−3
χ_{50}	315	315	−315	−21	−21	21	−21	−21	21	3	3	−21	21	3	3	−3	3	−3
χ_{51}	30	−2	0	14	−2	0	14	−2	0	6	−2	−2	0	6	−2	0	−2	0
χ_{52}	90	−6	0	42	−6	0	−6	10	0	2	10	−6	0	−6	−6	0	2	0
χ_{53}	90	−6	0	42	−6	0	−6	10	0	2	10	−6	0	−6	−6	0	2	0
χ_{54}	180	−12	0	84	−12	0	36	4	0	20	4	−12	0	12	−12	0	−4	0
χ_{55}	210	−14	0	98	−14	0	34	−14	0	10	−14	2	0	18	2	0	−6	0
χ_{56}	210	−14	0	98	−14	0	2	18	0	10	18	−14	0	−6	−14	0	2	0
χ_{57}	210	−14	0	98	−14	0	−14	2	0	−6	2	2	0	−6	2	0	2	0
χ_{58}	240	−16	0	112	−16	0	16	16	0	16	16	−16	0	0	−16	0	0	0
χ_{59}	420	−28	0	196	−28	0	20	−12	0	4	−12	4	0	12	4	0	−4	0
χ_{60}	630	−42	0	294	−42	0	−42	6	0	−18	6	6	0	−18	6	0	6	0
χ_{61}	630	−42	0	294	−42	0	6	−10	0	−2	−10	6	0	6	6	0	−2	0
χ_{62}	210	−14	0	−14	2	0	42	−6	−8	−6	2	−6	8	−6	2	0	2	0
χ_{63}	210	−14	0	−14	2	0	42	−6	8	−6	2	−6	−8	−6	2	0	2	0
χ_{64}	210	−14	0	−14	2	0	−6	10	−8	10	−14	−6	8	−6	2	0	2	0
χ_{65}	210	−14	0	−14	2	0	−6	10	8	10	−14	−6	−8	−6	2	0	2	0
χ_{66}	420	−28	0	−28	4	0	36	4	−16	4	−12	−12	16	−12	4	0	4	0
χ_{67}	420	−28	0	−28	4	0	36	4	16	4	−12	−12	−16	−12	4	0	4	0
χ_{68}	630	−42	0	−42	6	0	30	14	−24	−18	6	−18	24	6	6	0	−2	0
χ_{69}	630	−42	0	−42	6	0	30	14	24	−18	6	−18	−24	6	6	0	−2	0
χ_{70}	630	−42	0	−42	6	0	−18	30	−24	−2	−10	−18	24	6	6	0	−2	0
χ_{71}	630	−42	0	−42	6	0	−18	30	24	−2	−10	−18	−24	6	6	0	−2	0
χ_{72}	1260	−84	0	−84	12	0	60	−36	0	−4	12	12	0	−12	−4	0	4	0
χ_{73}	1260	−84	0	−84	12	0	−36	−4	0	28	−20	12	0	−12	−4	0	4	0
χ_{74}	1260	−84	0	−84	12	0	−36	−4	0	−4	12	12	0	12	−4	0	−4	0
χ_{75}	1260	−84	0	−84	12	0	−36	−4	0	−4	12	12	0	12	−4	0	−4	0

TABLE 7. The character table of $\overline{G} = 2^{4+5}:GL(4,2)$.

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Table 6 (continued)

$[g_i]_{GL(4,2)}$	$4B$						$5A$		$6A$		$6B$						$7A$					$7B$					$15A$		$15B$	
$[g_{ij}]_{\overline{G}}$	$4y$	$8b$	$4z$	$8c$	$8d$	$8e$	$5a$	$10a$	$6f$	$6g$	$6h$	$6i$	$12b$	$12c$	$12d$	$12e$	$7a$	$14a$	$14b$	$14c$	$28a$	$7b$	$14d$	$14e$	$14f$	$28b$	$15a$	$30a$	$15b$	$30b$
$\lvert C_{\overline{G}}(g_{ij})\rvert$	64	64	64	64	32	32	60	60	24	24	48	48	48	48	24	24	56	56	28	28	28	56	56	28	28	28	30	30	30	30
$\hookrightarrow 2^5{:}GL(5,2)$	$4I$	$8A$	$4J$	$8A$	$8B$	$8C$	$5A$	$10A$	$6E$	$6F$	$6C$	$6D$	$12A$	$12A$	$12B$	$12C$	$7A$	$14A$	$14A$	$14C$	$28A$	$7B$	$14B$	$14B$	$14D$	$28B$	$15B$	$30B$	$15A$	$30A$
χ_1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
χ_2	1	1	1	1	1	1	2	2	0	0	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1
χ_3	0	0	0	0	0	0	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1
χ_4	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	−1	−1	−1	−1	−1	−1	−1	−1	−1	−1	0	0	0	0
χ_5	−1	−1	−1	−1	−1	−1	1	1	−2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1
χ_6	−1	−1	−1	−1	−1	−1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	C	$\overline{C}$	$\overline{C}$
χ_7	−1	−1	−1	−1	−1	−1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{C}$	$\overline{C}$	C	C
χ_8	0	0	0	0	0	0	−2	−2	1	1	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	1	1	1	1
χ_9	−1	−1	−1	−1	−1	−1	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{10}	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	A	A	A	A	A	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	0	0	0	0
χ_{11}	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	$\overline{A}$	A	A	A	A	A	0	0	0	0
χ_{12}	0	0	0	0	0	0	1	1	0	0	−1	−1	−1	−1	−1	−1	0	0	0	0	0	0	0	0	0	0	1	1	1	1
χ_{13}	0	0	0	0	0	0	−1	−1	0	0	0	0	0	0	0	0	1	1	1	1	1	1	1	1	1	1	−1	−1	−1	−1
χ_{14}	0	0	0	0	0	0	0	0	−1	−1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{15}	1	1	−1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	−1	1	1	−1	1	−1	1	1	−1	1	−1	1	−1	1	−1
χ_{16}	1	1	−1	−1	1	−1	2	−2	0	0	−1	1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1
χ_{17}	0	0	0	0	0	0	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1
χ_{18}	0	0	0	0	0	0	0	0	1	−1	1	−1	1	−1	1	−1	−1	−1	1	−1	1	−1	−1	1	−1	1	0	0	0	0
χ_{19}	−1	−1	1	1	−1	1	1	−1	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1
χ_{20}	−1	−1	1	1	−1	1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	C	$-C$	$-\overline{C}$	$\overline{C}$
χ_{21}	−1	−1	1	1	−1	1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{C}$	$-\overline{C}$	$-C$	$\overline{C}$
χ_{22}	0	0	0	0	0	0	−2	2	1	−1	−1	1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1
χ_{23}	−1	−1	1	1	−1	1	0	0	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{24}	1	1	−1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	B	B	$-B$	B	$-B$	$\overline{B}$	$\overline{B}$	$-\overline{B}$	$\overline{B}$	$-\overline{B}$	0	0	0	0
χ_{25}	1	1	−1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	$\overline{B}$	$\overline{B}$	$-\overline{B}$	$\overline{B}$	$-\overline{B}$	B	B	$-B$	B	$-B$	0	0	0	0
χ_{26}	0	0	0	0	0	0	1	−1	0	0	−1	1	−1	1	−1	1	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1
χ_{27}	0	0	0	0	0	0	−1	1	0	0	0	0	0	0	0	0	1	1	−1	1	−1	1	1	−1	1	−1	−1	1	−1	1
χ_{28}	0	0	0	0	0	0	0	0	−1	1	1	−1	1	−1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{29}	1	1	1	1	−1	−1	0	0	0	0	1	1	1	1	−1	−1	1	1	1	−1	−1	1	1	1	−1	−1	0	0	0	0
χ_{30}	1	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	B	B	B	$-B$	$-B$	$\overline{B}$	$\overline{B}$	$\overline{B}$	$-\overline{B}$	$-\overline{B}$	0	0	0	0
χ_{31}	1	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	$\overline{B}$	$\overline{B}$	$\overline{B}$	$-\overline{B}$	$-\overline{B}$	B	B	B	$-B$	$-B$	0	0	0	0
χ_{32}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	1	1	−1	−1	−1	1	1	0	0	0	0
χ_{33}	−1	−1	−1	−1	1	1	0	0	0	0	−1	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{34}	−1	−1	−1	−1	1	1	0	0	0	0	1	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{35}	1	1	1	1	−1	−1	0	0	0	0	−1	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{36}	0	0	0	0	0	0	0	0	0	0	−1	−1	−1	−1	1	1	1	1	1	−1	−1	1	1	1	−1	−1	0	0	0	0
χ_{37}	0	0	0	0	0	0	0	0	0	0	1	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{38}	−1	−1	−1	−1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{39}	1	1	1	1	−1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{40}	1	1	−1	−1	−1	1	0	0	0	0	1	−1	1	−1	−1	1	1	1	−1	−1	1	1	1	−1	−1	1	0	0	0	0
χ_{41}	1	1	−1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	A	A	$-A$	$-A$	A	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-\overline{A}$	$\overline{A}$	0	0	0	0
χ_{42}	1	1	−1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	$\overline{A}$	$\overline{A}$	$-\overline{A}$	$-\overline{A}$	$\overline{A}$	A	A	$-A$	$-A$	A	0	0	0	0
χ_{43}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−1	−1	1	1	−1	−1	−1	1	1	−1	0	0	0	0
χ_{44}	−1	−1	1	1	1	−1	0	0	0	0	−1	1	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{45}	−1	−1	1	1	1	−1	0	0	0	0	1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{46}	1	1	−1	−1	−1	1	0	0	0	0	−1	1	−1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{47}	0	0	0	0	0	0	0	0	0	0	−1	1	−1	1	1	−1	1	1	−1	−1	1	1	1	−1	−1	1	0	0	0	0
χ_{48}	0	0	0	0	0	0	0	0	0	0	1	−1	1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{49}	1	1	−1	−1	−1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{50}	−1	−1	1	1	1	−1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{51}	2	−2	0	0	0	0	0	0	0	0	2	0	−2	0	0	0	2	−2	0	0	0	2	−2	0	0	0	0	0	0	0
χ_{52}	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	B	$-B$	0	0	0	$\overline{B}$	$-\overline{B}$	0	0	0	0	0	0	0
χ_{53}	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	$\overline{B}$	$-\overline{B}$	0	0	0	B	$-B$	0	0	0	0	0	0	0
χ_{54}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	−2	2	0	0	0	−2	2	0	0	0	0	0	0	0
χ_{55}	−2	2	0	0	0	0	0	0	0	0	−2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{56}	−2	2	0	0	0	0	0	0	0	0	2	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{57}	2	−2	0	0	0	0	0	0	0	0	−2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{58}	0	0	0	0	0	0	0	0	0	0	−2	0	2	0	0	0	2	−2	0	0	0	2	−2	0	0	0	0	0	0	0
χ_{59}	0	0	0	0	0	0	0	0	0	0	2	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{60}	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{61}	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{62}	0	0	−2	2	0	0	0	0	0	0	0	−2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{63}	0	0	2	−2	0	0	0	0	0	0	0	2	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{64}	0	0	2	−2	0	0	0	0	0	0	0	−2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{65}	0	0	−2	2	0	0	0	0	0	0	0	2	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{66}	0	0	0	0	0	0	0	0	0	0	0	2	0	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{67}	0	0	0	0	0	0	0	0	0	0	0	−2	0	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{68}	0	0	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{69}	0	0	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{70}	0	0	−2	2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{71}	0	0	2	−2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{72}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{73}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{74}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
χ_{75}	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

where in Table 6, $A = -\frac{1}{2} - \frac{\sqrt{7}}{2}i$, $B = -1 - \sqrt{7}i$ and $C = -\frac{1}{2} - \frac{\sqrt{15}}{2}i$.

After all, we applied the Clifford-Fischer theory to compute the character table of $\overline{G}$, which is a 75×75 complex matrix partitioned into 84 parts correspond to the 14 conjugacy classes of $GL(4,2)$ and the 6 inertia factor groups. In [15], the first two authors determined the character table of the other largest maximal subgroup of the affine general linear group $2^5{:}GL(5,2)$ of $GL(6,2)$, namely the group $2_+^{1+8}{:}GL(4,2)$, where they used similar techniques. The methods of the coset analysis and the Clifford-Fischer theory can also be applied to the other maximal subgroups of the affine general linear group $2^5{:}GL(5,2)$ of $GL(6,2)$, where the authors have already started working on some of these groups.

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