Iranian Journal of Mathematical Sciences and Informatics
Vol. 14, No. 2 (2019), pp 69-77
DOI: 10.7508/ijmsi.2019.02.007

The Study of Some Boundary Value Problems Including Fractional Partial Differential Equations with non-Local Boundary Conditions

R. Hosseini [a,*], M. Jahanshahi[a] ,A.A.Pashavand [b], N.Aliev[b]

[a]Department of Mathematics, Azarbaijan Shahid Madani University, 35 Km Tabriz-Maraghe Road, Tabriz, Iran.
[b]Institute of Mathematics and Mechanics of NAS of Azarbijan, Baku, Azarbijan.

E-mail: hosseini-k@azaruniv.edu
E-mail: Jahanshahi@azaruniv.edu
E-mail: apashavand@yahoo.com
E-mail: nihan@aliev.info

Abstract. In this paper, we consider some boundary value problems (BVP) for fractional order partial differential equations (FPDE) with non-local boundary conditions. The solutions of these problems are presented as series solutions analytically via modified Mittag-Leffler functions. These functions have been modified by authors such that their derivatives are invariant with respect to fractional derivative. The presented solutions for these problems are as infinite series. Convergence of series solutions and uniqueness of them are established by general theory of mathematical analysis and theory of ODEs.

Keywords: Mittag-Leffler function, Fractional partial differential equation, non-Local boundary condition.

2000 Mathematics Subject Classification: 35A99 .

*Corresponding Author

Received 13 March 2017; Accepted 30 December 2017
©2019 Academic Center for Education, Culture and Research TMU

[Downloaded from ijmsi.com on 2026-11-23]

1. Introduction

Fractional calculus and fractional differential equations have improved more in recent years [16,17]. Moreover, regarding the integral representation of the solutions of fractional differential equations, these solutions are not very computational, because their integral represnetations have complex integrands, especially include terms of Gamma function [3,11].
In this paper, we will search the form of solutions for FPDE as series solutions by modified Mittag-Leffler functions. These functions are invariant with respect to fractional order as same as Euler exponential function to ordinary derivative. The solutions are presented in this paper are based on Mittag-Lefller functions which are very computational [1,2]. One can constructs an approxmiate solution by choosing specific number of terms from series solution. The general modified Mittag- Leffler function can be written in the following form [11,12,13]:

$$h_p(x,r)=\sum_{k=1}^{\infty}\frac{r^k x^{kp-1}}{(kp-1)!}, \tag{1.1}$$

where the real number p is the fractional step derivative and r is and unknown parameter. It is easy to see that, the following relation is valid

$$D^{np}h_p(x,r)=r^n.h_p(x,r), \tag{1.2}$$

similar to exponential function $D^n e^{rx}=r^n e^{rx}$. The parameter r is usually root of characteristic equation. For example we consider the following fractional ordinary differential equation:

$$y^{(\frac{2}{3})}(x)-3y^{(\frac{1}{3})}(x)+2y(x)=0, \tag{1.3}$$

note that $p=\frac{1}{3}$, $np=\frac{2}{3}$ and $n=2$, we consider the following relations:

$$D^{\frac{1}{3}}h_{\frac{1}{3}}(x,r)=rh_{\frac{1}{3}}(x,r),\quad D^{\frac{2}{3}}h_{\frac{1}{3}}(x,r)=r^2h_{\frac{1}{3}}(x,r). \tag{1.4}$$

After replacing the above amounts in the fractional equation (2) we have:

$$r^2h_{\frac{1}{3}}(x,r)-3rh_{\frac{1}{3}}(x,r)+2h_{\frac{1}{3}}(x,r)=0, \tag{1.5}$$

and characteristic equation will be $r^2-3r+2=0$. The roots of this equation are $r_1=1, r_2=2$. Then the general solution of equation (2) is as follows:

$$y(x)=c_1h_{\frac{1}{3}}(x,1)+c_2h_{\frac{1}{3}}(x,2) \tag{1.6}$$

$$=c_1\sum_{k=1}^{\infty}\frac{(1)^k x^{(\frac{k}{3})-1}}{((\frac{k}{3})-1)!}+c_2\sum_{k=1}^{\infty}\frac{(2)^k x^{(\frac{k}{3})-1}}{((\frac{k}{3})-1)!}. \tag{1.7}$$

[Downloaded from ijmsi.com on 2026-11-23]

2. BVP for FPDE on Unbounded Domain

In this section, we consider some BVP for FPDE in unbounded domain. For this, consider the following problem in the first quarter:

$$D_x^\alpha u(x,y) = D_y^\alpha u(x,y), \quad x>0, y>0, \tag{2.1}$$

with non-local boundary condition

$$u(t,0) = au(0,t) + \varphi(t), \quad t \geqslant 0, \tag{2.2}$$

where $\alpha \in (0,1)$ is fractional order and $\varphi(t)$ is a real valued smooth function and a is real constant. According to the introduced modified Mittag-Leffler function the solution of equation (2.1) is looking for as following series:

$$u(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} u_{mn}\frac{x^{m\alpha}}{(m\alpha)!}\frac{y^{n\alpha}}{(n\alpha)!}, \tag{2.3}$$

where u_{mn} are unknown coefficients. Let's suppose $(-k\alpha)! = \infty$, for $k \in \mathbb{N}$. According to the α-order derivative of Mittag-Leffler function, we have

$$D_x^\alpha u(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} u_{mn}\frac{x^{(m-1)\alpha}}{[(m-1)\alpha]!}\frac{y^{n\alpha}}{(n\alpha)!} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} u_{(m+1)n}\frac{x^{m\alpha}y^{n\alpha}}{(m\alpha)!(n\alpha)!}, \tag{2.4}$$

and

$$D_y^\alpha u(x,y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} u_{mn}\frac{x^{(m\alpha)}}{(m\alpha)!}\frac{y^{(n-1)\alpha}}{[(n-1)\alpha]!} = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty} u_{m(n+1)}\frac{x^{m\alpha}y^{n\alpha}}{(m\alpha)!(n\alpha)!}. \tag{2.5}$$

By replacing these amounts in equation (2.1), we get the following recurrence (difference) equation:

$$u_{(m+1)n} = u_{m(n+1)}, \quad m,n \geqslant 0, \tag{2.6}$$

by imposing boundary conditions (2.2) to series solution (2.3) we obtain:

$$\sum_{m=0}^{\infty} u_{m0}\frac{t^{m\alpha}}{(m\alpha)!} = a\sum_{n=0}^{\infty} u_{0n}\frac{t^{n\alpha}}{(n\alpha)!} + \varphi(t), \quad t \geqslant 0, \tag{2.7}$$

or

$$\sum_{k=0}^{\infty}(u_{k0} - au_{0k})\frac{t^{k\alpha}}{(k\alpha)!} = \varphi(t), \quad t \geqslant 0, \tag{2.8}$$

from the difference equation (2.4) we see that

$$u_{mn} = u_{(m+n)0} = u_{0(m+n)}, \quad m,n \geqslant 0, \tag{2.9}$$

by using (2.6) and (2.5) we have:

$$\sum_{k=0}^{\infty} u_{k0}(1-a)\frac{t^{k\alpha}}{(k\alpha)!} = \varphi(t), \tag{2.10}$$

[Downloaded from ijmsi.com on 2026-11-23]

or

$$\sum_{k=0}^{\infty} u_{k0}\frac{t^{k\alpha}}{(k\alpha)!} = \frac{\varphi(t)}{1-\alpha}, \quad t \geqslant 0, \tag{2.11}$$

specially, for $t = 0, u_{00} = \frac{\varphi(0)}{1-\alpha}$. From (2.7) for some finite values of the fractional derivative of $k \in \mathbb{N}$, the $\alpha-$fractional derivative terms get

$$u_{00}\frac{t^{-\alpha}}{(-\alpha)!} + u_{10}\frac{1}{0!} + u_{20}\frac{t^{\alpha}}{\alpha!} + u_{30}\frac{t^{2\alpha}}{(2\alpha)!} + ... = \frac{1}{1-\alpha}D^{\alpha}\varphi(t), \tag{2.12}$$

specially for $t = 0$, we have:

$$u_{10} = \frac{1}{1-\alpha}D^{\alpha}\varphi(t)|_{t=0}, \tag{2.13}$$

by continuing this process, we get the following relation

$$u_{k0} = \frac{1}{1-\alpha}D^{k\alpha}\varphi(t)|_{t=0}, \quad k \geqslant 0. \tag{2.14}$$

To sum up, we conclude the following theorem:

Theorem 2.1. *Suppose a, α are real known constants and $\alpha \in (0, 1)$. Also $\varphi(t)$ is a infinite differentiable function and satisfies in the following condition:*

$$\exists \quad 0 < M \in \mathbb{R}; \quad \left|D^{k\alpha}\varphi(t)|_{t=0}\right| \leqslant M, \quad k \geqslant 0. \tag{2.15}$$

Then the problem (2.1)-(2.2) has a unique solution in the form of (2.3).

Remark 2.2. According to M-test Weierstrass, convergence of series solution

$$u(x, y) = \sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{1}{1-\alpha}D^{(m+n)\alpha}\varphi(t)|_{t=0}\frac{x^{m\alpha}}{(m\alpha)!}\frac{y^{n\alpha}}{(n\alpha)!}, \tag{2.16}$$

is guaranteed, because its majorant series is converge for all fixed values of $x > 0$ and $y > 0$, that is

$$|u(x, y)| \leq \frac{M}{1-\alpha}\sum_{m=0}^{\infty}\frac{x^{m\alpha}}{(m\alpha)!}\sum_{n=0}^{\infty}\frac{x^{n\alpha}}{(n\alpha)!}, \tag{2.17}$$

is converge. Because of factorial terms $(m\alpha)!$ and $(n\alpha)!$ in denominator.

3. BVP for FPDE in Bounded Domain

We study the following BVP for FPDE in a bounded domain as follows:

$$D_x^{\sqrt{3}}u(x, y) = D_y^{\sqrt{2}}u(x, y), \quad x \in (0, 1), y \in (0, 1), \tag{3.1}$$

with non-local boundary condition:

$$\alpha_0 u(0, t) + \alpha_1 u(1, t) + \beta_0(t, 0) + \beta_1 u(t, 1) = \varphi(t); \quad t \in [0, 1], \tag{3.2}$$

α_i and $\beta_i, i = 0, 1$ are known real constants and $\varphi(t)$ is known real valued continuous function. According to modified Mittag-Leffler function in section

[Downloaded from ijmsi.com on 2026-11-23]

1, we search the solution of the equation (3.1) in the form of the following series:

$$u(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}u_{mn}\frac{x^{-1+(m-1)\sqrt{3}}}{[-1+(m-1)\sqrt{3}]!}\frac{y^{-1+n\sqrt{2}}}{(-1+n\sqrt{2})!}. \tag{3.3}$$

Note that the function

$$h_{\frac{1}{n}}(x)=\frac{x^{\frac{1-n}{n}}}{(\frac{1-n}{n})!}+\frac{x^{\frac{2-n}{n}}}{(\frac{2-n}{n})!}+...=\sum_{k=1}^{\infty}\frac{x^{\frac{k-n}{n}}}{(\frac{k-n}{n})!},\quad (-1)!=\infty, \tag{3.4}$$

is invariant to the fractional order $\alpha=\frac{1}{n}$ step derivative. Therefore we have

$$D_x^{\sqrt{3}}u(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}u_{mn}\frac{x^{-1+(m-1)\sqrt{3}}}{[-1+(m-1)\sqrt{3}]!}\frac{y^{-1+n\sqrt{2}}}{(-1+n\sqrt{2})!} \tag{3.5}$$

$$=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}u_{(m+1)n}\frac{x^{-1+m\sqrt{3}}}{[-1+m\sqrt{3}]!}\frac{y^{-1+n\sqrt{2}}}{(-1+n\sqrt{2})!}, \tag{3.6}$$

and

$$D_y^{\sqrt{2}}u(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}u_{m(n+1)}\frac{x^{-1+m\sqrt{3}}}{[-1+m\sqrt{3}]!}\frac{y^{-1+n\sqrt{2}}}{(-1+n\sqrt{2})!}. \tag{3.7}$$

By replacing these amounts in the equation(3.1), we get the following recurrence relation

$$u_{(m+1)n}=u_{m(n+1)}\quad ,m,n=1,2,... \tag{3.8}$$

or we have

$$u_{mn}=u_{m+n-1},\quad m,n=1,2,.... \tag{3.9}$$

By imposing the boundary condition (3.2) in a simple case that, $\alpha_0=\beta_0=0$.

$$\alpha_1u(1,t)+\beta_1u(t,1)=\varphi(t),\quad \alpha_1,\beta_1\in\mathbb{R}, \tag{3.10}$$

we obtain the following relation:

$$\alpha_1\sum_{k=1}^{\infty}u_{k1}\sum_{m=1}^{k}\frac{1}{(-1+m\sqrt{3})!}\frac{t^{-1+(k+1-m)\sqrt{2}}}{[-1+(k+1-m)\sqrt{2}]!}+ \tag{3.11}$$

$$\beta_1\sum_{k=1}^{\infty}u_{k1}\sum_{m=1}^{k}\frac{t^{-1+m\sqrt{3}}}{(-1+m\sqrt{3})!}\frac{1}{[-1+(k+1-m\sqrt{2}]!}=\varphi(t), \tag{3.12}$$

or

$$\sum_{k=1}^{\infty}u_{k1}v_k(t)=\varphi(t),\quad t\in[0,1], \tag{3.13}$$

where

$$v_k(t)=\sum_{m=1}^{\infty}\{\frac{\alpha_1}{-1+m\sqrt{3}}\frac{t^{-1+(k+1-m)\sqrt{2}}}{[-1+(k+1-m)\sqrt{2}]} \tag{3.14}$$

$$+\frac{\beta_1}{[-1+(k+1-m)\sqrt{2}]}\frac{t^{-1+m\sqrt{3}}}{-1+m\sqrt{3}}\},\quad k=1,2,.... \tag{3.15}$$

[Downloaded from ijmsi.com on 2026-11-23]

Suppose the function $w_k(t)$ is biorthogonal to $v_k(t)$, that is:

$$(v_k, w_s) = \delta_{ks} = \begin{cases} 1, & k = s \\ 0, & k \neq s \end{cases} \quad k, s = 1, 2, ..., \tag{3.16}$$

then from (3.5) and (3.6) we will have

$$\sum_{k=1}^{\infty} u_{k1}(v_k, w_s) = (\varphi, w_s), \quad s = 1, 2, ... \tag{3.17}$$

and also $u_{s1} = (\varphi, w_s), s = 1, 2, ...$. Finaly by using (3.7), we get the following series for the solution of problem (3.1)-(3.2):

$$u(x, y) = \sum_{k=1}^{\infty} (\varphi, w_s) \sum_{m=1}^{k} \frac{x^{-1+m\sqrt{3}}}{(-1+m\sqrt{3})!} \frac{y^{-1+(k+1-m)\sqrt{2}}}{[-1+(k+1-m)\sqrt{2}]!}. \tag{3.18}$$

To sum up we conclude the following theorem:

Theorem 3.1. *If α_1, β_1 are real constants, and $\varphi(t)$ is a continuous real valued function, and the functions $w_s(t)$ are biorthogonal to $v_k(t)$ in the form of (3.6), then the BVP (3.1)-(3.4) has a unique solution in form of (3.8).*

4. Analogy of Fractional Cauchy-Riemann Equation

In this section, we consider a FPDE which it is the fractional analogy of the Cauchy-Riemann equation. In fact, this equation is derived from decomposition of Laplace equation

$$(\frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_1^2})u(x) = (\frac{\partial}{\partial x_2} + i\frac{\partial}{\partial x_1})(\frac{\partial}{\partial x_2} - i\frac{\partial}{\partial x_1})u(x), \tag{4.1}$$

and

$$(\frac{\partial}{\partial x_2} + i\frac{\partial}{\partial x_1})u(x) = (D_2^{\frac{1}{2}} + \frac{1-i}{\sqrt{2}} D_1^{\frac{1}{2}})(D_2^{\frac{1}{2}} - \frac{1-i}{\sqrt{2}} D_1^{\frac{1}{2}})u(x). \tag{4.2}$$

Therefore we consider a BVP consists of the analogy of fractional Cauchy-Riemann equation

$$D_2^{\frac{1}{2}} u(x) + a D_1^{\frac{1}{2}} u(x) = 0, \quad 0 < x_j < 1, j = 1, 2, \tag{4.3}$$

with boundary condition

$$u(t, 0) + \alpha u(0, t) + \beta u(t, 1) + \gamma u(1, t) = \varphi(t), \quad 0 \leq t \leq 1, \tag{4.4}$$

where α, β, γ, are complex constants and $\varphi(t) \in \mathbb{C}[0, 1]$. We search the solution of equation (4.1) in the following form

$$u(x) = u(x_1, x_2) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} u_{mn} \frac{x_1^{\frac{m}{2}}}{(\frac{m}{2})!} \frac{x_2^{\frac{n}{2}}}{(\frac{n}{2})!}. \tag{4.5}$$

[Downloaded from ijmsi.com on 2026-11-23]

We suppose $(\frac{-1}{2})! = \infty$. Then by derivating of proposed solution (4.3) and considering some relations in pervious sections we will have the recurrence relation

$$u_{m(n+1)} + au_{(m+1)n} = 0, \quad m \geqslant 0, n \geqslant 0, \tag{4.6}$$

or

$$u_{m(n+1)} = (-a)^n u_{(m+1)n}, \quad m \geqslant 0, n \geqslant 0. \tag{4.7}$$

By substituting in boundary condition (4.2) we get:

$$\sum_{m=0}^{\infty} u_{m0} \frac{t^{\frac{m}{2}}}{(\frac{m}{2})!} + \alpha \sum_{n=0}^{\infty} u_{0n} \frac{t^{\frac{n}{2}}}{(\frac{n}{2})!} + \sum_{m=0}^{\infty} \frac{t^{\frac{m}{2}}}{(\frac{m}{2})!} \sum_{n=0}^{\infty} u_{mn} \frac{1}{(\frac{n}{2})!} \tag{4.8}$$

$$+ \sum_{n=0}^{\infty} \frac{t^{\frac{n}{2}}}{(\frac{n}{2})!} \sum_{m=0}^{\infty} u_{mn} \frac{1}{(\frac{m}{2})!} = \varphi(t). \tag{4.9}$$

Suppose the function $\varphi(t)$, has a series expansion as follows:

$$\varphi(t) = \sum_{k=0}^{\infty} \varphi_k \frac{t^{\frac{k}{2}},}{(\frac{k}{2})!} \tag{4.10}$$

then we have:

$$[1 + \alpha(-a)^m] u_{m0} \tag{4.11}$$

$$+ \sum_{n=0}^{\infty} \frac{1}{(\frac{n}{2})!} [\beta(-a)^n + \gamma(-a)^m] u_{(m+n)0} = \varphi_m, \quad m \geqslant 0, \tag{4.12}$$

If the coefficient of u_{m0} is unvanish, then we can determine the arbitrary coefficients of u_{k0} of series solution (4.3). Finally we will have the following final theorem.

Theorem 4.1. *If α, β, γ are complex constsnts and $\varphi(t)$ is complex valued function, and the expansion (4.5) and the condition*

$$1 + \alpha(-a)^m + \beta + \gamma(-a)^m \neq 0, \quad m \geqslant 0, \tag{4.13}$$

are valid, then the BVP (4.1)-(4.2) has a unique solution in form of (4.3)

Remark 4.2. In the special case of boundary condition (4.2), $\beta = \gamma = 0$ then under condition

$$1 + \alpha(-a)^m \neq 0, \tag{4.14}$$

we get the following results:

$$u_{k0} = \frac{\varphi_k}{1 + \alpha(-a)^k}; \quad k \geqslant 0, \tag{4.15}$$

and the normal solution for the problem (4.1)-(4.2) is

$$u(x) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{x_1^{\frac{m}{2}}}{(\frac{m}{2})!} \frac{x_2^{\frac{n}{2}}}{(\frac{n}{2})!} (-a)^n \frac{\varphi_{m+n}}{1 + \alpha(-a)^{m+n}}. \tag{4.16}$$

[Downloaded from ijmsi.com on 2026-11-23]

Therefore this formal solution is converge when the following conditions are satisfied:

$$|a| \leq 1, |\varphi_k| < M, \quad k \geqslant 0. \tag{4.17}$$

5. Conclusion

In this paper, some BVP involving FPDE were presented. By means of modified Mittag-Leffler functions we presented their solutions in the form of series. Then by using the theory of ordinary differential equations and the concept of fractional step, we extended the solving process to the general and advanced cases. Finally, regarding the general terms of series solutions (1.4), (2.3), (3.8) and (4.3) which include factorial terms, the convergency of sereis solution is guaranteed. Conditions of existence and uniquencees of solution for problem (2.1)-(2.2) and problem (3.1)-(3.2) the as three theorem were expressed.

Acknowledgement

The authors are thankful to referees for their valuable suggestions.

References

1. A. Ahmadkhanlu, M. Jahanshahi, On well-posed of boundary value problems including fractional order differential equations, *Southeast Asian Bulletin of Mathematics*, **36**(1), (2013).
2. N.A. Aliyev, A.A. Pashavand, A boundary value problem for a fractional order ordinary linear differential equation with a constant coefficient, *Proceedings of the Institute of Applied Mathematics, Azerbaijan, Baku*, **4**(1), (2015), pp.3-7
3. D. Baleanu, H. Mohammadi, Sh. Rezapour, *Some existence results for a nonlinear fractional differential equation on partially ordered Banach spaces*, 2012.
4. A.H. Bhrawy, S.S. Ezz-Eldien, A new Legendre operational technique for delay fractional optimal control problems, *Calcolo*, **53**, (2016), 521-543.
5. K. Diethelm, *The analysis of fractional differential equations: an application-oriented exposition using differential operators of Caputo type*, in: Lecture Notes in Mathematics, Springer-Verlag, 2010.
6. K. Diethelm, An algorithm for the numerical solution of differential equations of fractional order, *Electronic Transactions on Numerical Analysis*, **5**, (1997), 16.
7. J.T. Edwards, N. J. Ford, A.C. Simpson, The numerical solution of linear multi-term fractional differential equations: systems of equations, *Journal of Computational and Applied Mathematics*, **148**(2), (2002), 401418.
8. S.S. Ezz-Eldien, New quadrature approach based on operational matrix for solving a class of fractional variational problems, *J. Comput. Phys.*, **317**, (2016), 362-381.
9. S.S. Ezz-Eldien, A. A. El-Kalaawy, Numerical simulation and convergence analysis of fractional optimization problems with right-sided Caputo fractional derivative, *J. Comput. Nonlinear Dyn.*, (2017), DOI:10.1115/1.4037597.
10. S. S. Ezz-Eldien, A. H. Bhrawy, A.A. El-Kalaawy, Direct numerical technique for isoperimetric fractional variational problems based on operational matrix, *J. Vib. Control*, (2017), DOI: 10.1177/1077546317700344.

[Downloaded from ijmsi.com on 2026-11-23]

11. A. A. Kilbas, M. Saigo, R. K. Saxena, Solution of volterra integrodifferential equations with generalized Mittag-Leffler function in the kernels, *J. Integral Equations and Applications*, **14**, (2002), 377-396.
12. A. A. Kilbas, H. M. Srivastava, J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*,**204**, Elsevier, 2006.
13. A. A. Kilbas, M. Saigo, R. K. Saxena, Generalized mittag-leffler function and generalized fractional calculus operators, *Integral Transforms and Special Functions*, **15**, (2004), 31-49.
14. V. Lakshmikantham, A. S. Vatsala, Theory of fractional differential inequalities and applications, *Communications in Applied Analysis*, **11**, (2007), 395402.
15. D. Nazari Susahab, S. Shahmorad, M. Jahanshahi, Efficient quadrature rules for solving nonlinear fractional integro-differential equations of the Hammerstein type, *Applied Mathematical Modeling*, **39**, (2015), 54525458.
16. S. G. Samko, A. A. Kilbas, Q.I. Marichev, *Fractional Integrals and Derivatives; Theory and Applications*, Gordan and Breach, Amsterdam, 1993.
17. I. Podlubny,*Fractional Differential Equations*, Academic Press, California, 1999.

[Downloaded from ijmsi.com on 2026-11-23]