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Design of robust controller by neuro-fuzzy system in a prescribed region via state feedback

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Abstract. In this paper, first a new algorithm for pole assignment of closed-loop multi-variable controllable systems in a prescribed region of the z-plane is presented. Then, robust state feedback controllers are designed by implementing a neural fuzzy system for the placement of closed-loop poles of a controllable system in a prescribed region in the left-hand side of z-plane. A new method based on the parameterizations of condition number function of a closed-loop system whose poles are varied in a prescribed region by neural fuzzy system is also designed.

Keywords: Parametric controllers,Neural fuzzy system,Pole placement,State feedback.

2000 Mathematics subject classification: 93xx.

1. Introduction

Pole placement, also known as pole assignment, is an essential problem in order to obtain stability and robustness in the control system theory [1], [3], [5], [11], [13]. In many problems, pole assignment in a prescribed region is the main goal instead of assignment of a certain pole spectrum to the problem. For example, we can allude to, the pole assignment into the prescribed region

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of the left-half z-plane for continuous systems and inside unit disk for discrete systems(see Figure 1), in order to stabilize a controllable system. The other important concept, in pole assignment problem, is to decrease sensitivity or to increase robustness of the closed-loop matrix to external perturbations [4], [11], [14], [17], [18]. In this paper, we have three sections. In the first section a new explicit method, based on the parameterizations of gain matrix, for pole assignment in a prescribed region for continuous systems is described. This method is to look at implicit methods of placing poles in regions instead of points. Arcasoy [2] has extendedKalman's criterion of a single state optimal control system to the multivariable case and later he introduced an algorithm, which obtains an optimal regulator with the prescribed pole assignment property using frequency response approach. In this work, a new method is given for determination of feedback controller matrix K for placing poles in prescribed regions for multivariable time-invariant continuous systems by taking advantage of non-linear parametric forms proposed by Karbassi and Tehrani [10]. In the second section,by implementing the neuro-fuzzy hybrid system, as a universal approximation of the robustness function, and by using the method described in the first section, we determine the poles spectrum with minimum sensitivity to external perturbations. Then by using an iterative method, assignment of linear independent eigenvectors corresponding to the required poles is obtained such that feedback matrix is insensitive to perturbations as far as possible. Finally a numerical example is presented to illustrate the advantages of this new method.

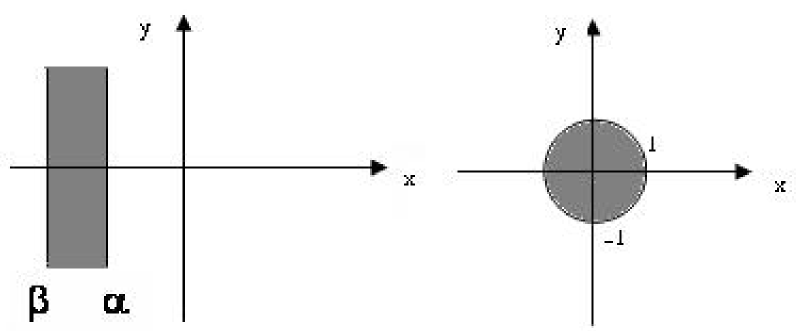

FIGURE 1. a: Specified regions for continuous systems b: Specified regions for discrete systems

2. Explicit method of pole placement in a prescribed region by state feedback

Consider a controllable linear time-invariant system defined by the state equation

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $x(k) \in R^n, u(k) \in R^m$ and the matrices A and B are real constant matrices of dimensions $n \times n$ and $n \times m$ respectively, with $\text{rank}(B) = m$. The aim of pole assignment is to design a state feedback controller,

$$u(t) = Kx(t) \tag{2}$$

where $K \in R^{m\times n}$ is the state feedback controller matrix, producing a closed-loop system

$$\dot{x}(t) = (A + BK)x(t) \tag{3}$$

with a satisfactory response by shifting controllable poles from actual to desirable locations.

Karbassi and Bell [9] have presented an algorithm obtaining an explicit parametric controller matrix K_α by performing elementary similarity operations, which transforms the controllable pair (B, A) into primary vector companion form

$$K_\alpha = K_p + F_\alpha \tag{4}$$

where K_p is the primary state feedback matrix such that the closed-loop matrix $(A + BK_p)$ has the required poles and $F_\alpha$is the parametric state feedback matrix with linear parameters such that $(A + BF_\alpha)$ has zero poles [9]. Later Karabassi and Tehrani [10], obtained an explicit parametric formula for K_α.Here we briefly review important results of [9]. Consider state transformation:

$$x(t) = T\tilde{x}(t) \tag{5}$$

where T can be obtained by elementary similarity operations as described in [18]. In this way, $\tilde{A} = T^{-1}AT$ and $\tilde{B} = T^{-1}B$ are in the compact canonical form known as vector companion form:

$$\tilde{A} = \begin{bmatrix} G_0 \\ I_{n-m}, 0_{n-m,m} \end{bmatrix}, \qquad \tilde{B} = \begin{bmatrix} B_0 \\ 0_{n-m,m} \end{bmatrix} \tag{6}$$

where G_0 is a $m \times n$ matrix and B_0 is an $m \times m$ upper triangular matrix. Note that if the Kronecker invariants [17] of the pair (B, A) are regular, then $\tilde{A}$ and

$\tilde{B}$ are always in the above form [7]. The state feedback matrix, which assigns all the poles to zero, for the transformed pair $(\tilde{B}, \tilde{A})$, is then chosen as

$$u = -B_0^{-1} G_0 \tilde{x} = \tilde{F}\tilde{x} \tag{7}$$

which results in the primary state feedback matrix for the pair (B, A) defined as

$$F_P = \tilde{F} T^{-1} \tag{8}$$

The transformed closed-loop matrix $\tilde{\Gamma}_0 = \tilde{A} + \tilde{B}\tilde{F}$ assumes a compact Jordan form with zero eigenvalues:

$$\tilde{\Gamma}_0 = \begin{bmatrix} 0_{m,n} \\ I_{n-m}, 0_{n-m,m} \end{bmatrix} \tag{9}$$

Now let $\tilde{A}_\lambda$ be any matrix in vector companion form, i.e.

$$\tilde{A}_\lambda = \begin{bmatrix} G_\lambda \\ I_{n-m}, 0_{n-m,m} \end{bmatrix} \tag{10}$$

with the eigenvalue spectrum $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_n\}$, a set of self conjugate eigenvalue. Then

$$\tilde{K} = B_0^{-1}(G_\lambda - G_0) \tag{11}$$

is the feedback matrix which assigns the pole spectrum to the closed-loop matrix $\tilde{\Gamma} = \tilde{A} + \tilde{B}\tilde{K}$. Furthermore, if K is the controller matrix which assigns the set of self conjugate poles $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_n\}$ to the transformed pair $(\tilde{B}, \tilde{A})$, then

$$K = \tilde{K} T^{-1} = B_0^{-1}(G_\lambda - G_0) T^{-1} \tag{12}$$

is the controller matrix which assigns the same set of poles to the pair (B, A). For more detail, the interested reader is referred to $[7 - 10]$. It is from this result that a general framework for obtaining the explicit form of parametric controllers with non-linear parameters can be deduced. Let

$$det(\tilde{A}_\lambda - \lambda I) = P_n(\lambda) = 0 \tag{13}$$

where

$$P_n(\lambda) = (-1)^n (\lambda^n + c_1 \lambda^{n-1} + ... + c_{n-1}\lambda + c_n) \tag{14}$$

is the characteristic polynomial of the closed-loop system. Since it is required that that zeros of this polynomial lie in the set $\Lambda = \{\lambda_1, \lambda_2, ..., \lambda_n\}$, it is clear that

$$P_n(\lambda) = (-1)^n (\lambda - \lambda_1)(\lambda - \lambda_2)...(\lambda - \lambda_n) \tag{15}$$

By equating these two equations the coefficients c_i,$(i = 1, 2, ..., n)$ can be obtained as follows [6]:

$$c_1 = -\sum_{i=1}^{n}(\lambda_i) = -trace(\tilde{A}_\lambda) \tag{16}$$

$$c_2 = \sum_{i,j=1,i\neq j}^{n}(\lambda_i\lambda_j) = -(c_1 trace(\tilde{A}_\lambda) + trace(\tilde{A}_\lambda^2))/2 \tag{17}$$

$$... \quad ... \quad ...$$

(18)

$$c_n = (-1)^n \Pi_{i=1}^{n}(\lambda_i) = -(c_{n-1}trace(\tilde{A}_\lambda)+c_{n-2}trace(\tilde{A}_\lambda^2)+...+c_1 trace(\tilde{A}_\lambda^{n-1}))/n$$

There is a MATLAB program that computes $c_1, c_2, ..., c_n$ when the desired roots of the characteristic polynomial, i.e. λ_i , $(i = 1, 2, ..., n)$ are specified. Now by direct computation of $\det(\tilde{A}_\lambda - \lambda I) = P_n(\lambda)$ in parametric form and equating the coefficients of the characteristic polynomial with $(16-18)$, a non-linear system of equations is obtained:

$$f_1(g_{11}, g_{12}, ...g_{1n}, g_{21}, g_{22}, ..., g_{m1}, g_{m2}, ..., g_{mn}) = c_1 \tag{19}$$

$$f_2(g_{11}, g_{12}, ...g_{1n}, g_{21}, g_{22}, ..., g_{m1}, g_{m2}, ..., g_{mn}) = c_2 \tag{20}$$

$$... \quad ... \quad ...$$

$$f_n(g_{11}, g_{12}, ...g_{1n}, g_{21}, g_{22}, ..., g_{m1}, g_{m2}, ..., g_{mn}) = c_n \tag{21}$$

where g_{ij}, $(i = 1, 2, ..., m), j = (1, 2, ..., n)$, are the elements of G_λ:

$$G_\lambda = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ . & . & \cdots & . \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix} \tag{22}$$

In this way, a non-linear system of n equations with $n \times m$ unknown is obtained. By choosing $N = n(m-1)$ unknowns arbitrarily it is then possible to solve the system. There are many ways to reduce the non-linear system of equation to a linear one. A simple choice is to let $g_{kj} = 1$ for$(k = 1, 2, ..., i-1, i+1, ...m) and (j = 1, 2, ..., n)$ in G_λ. That is to choose G_λ in the form:

$$G_\lambda = \begin{bmatrix} 1 & 1 & ... & 1 \\ . & . & ... & . \\ g_{i1} & g_{i2} & \cdots & g_{in} \\ . & . & ... & . \\ 1 & 1 & ... & 1 \end{bmatrix} \tag{23}$$

The $i-th$ row of G_λ can then be obtained by solving a linear system of equations of dimension n. Now, if it is required to locate the poles in a region specified by $x = \alpha$ and $x = \beta$ in the left half z-plane (as in figure 1:a) such that

$$\beta \leq real(\lambda_i) \leq \alpha \quad for\ i = 1, 2, ..., n \tag{24}$$

We may assume that all the poles are set equal to α or β. Then according to the definition of the characteristic equation

$$t_{11} \leq c_1 \leq t_{12} \tag{25}$$

where

$$t_{11} = min\{-n\alpha, -n\beta\} \tag{26}$$

and

$$t_{12} = max\{-n\alpha, -n\beta\} \tag{27}$$

are lower and upper bounds for c_1, respectively. Also

$$t_{k1} \leq c_k \leq t_{k2} \tag{28}$$

where

$$t_{k1} = min\{(-1)^k \begin{pmatrix} n \\ k \end{pmatrix} \alpha^k, (-1)^k \begin{pmatrix} n \\ k \end{pmatrix} \beta^k\} \tag{29}$$

and

$$t_{k2} = max\{(-1)^k \begin{pmatrix} n \\ k \end{pmatrix} \alpha^k, (-1)^k \begin{pmatrix} n \\ k \end{pmatrix} \beta^k\} \tag{30}$$

and finally

$$t_{n1} \leq c_n \leq t_{n2} \tag{31}$$

where

$$t_{n1} = min\{(-1)^n \alpha^n, (-1)^n \beta^n\} \tag{32}$$

and

$$t_{n2} = max\{(-1)^n \alpha^n, (-1)^n \beta^n\} \tag{33}$$

in which t_{j1} and t_{j2} are the lower and upper bounds of the $j-th$ coefficients. We now introduce parameter ω to ensure a convex combination of the lower and upper limits, that is:

$$C_1 = \omega t_{12} + (1-\omega) t_{11} \tag{34}$$

$$C_2 = \omega t_{22} + (1-\omega) t_{21} \tag{35}$$

$$... \quad ... \quad ...$$

$$C_n = \omega t_{n2} + (1-\omega) t_{n1} \tag{36}$$

where $\omega \in (0,1)$. Then the non-linear system of equations $(19)-(21)$ is reduced to a linear system with new right hand sides:

$$f_1(g_{11}, g_{12}, ...g_{1n}, g_{21}, g_{22}, ..., g_{m1}, g_{m2}, ..., g_{mn}) = C_1 \tag{37}$$

$$f_2(g_{11}, g_{12}, ...g_{1n}, g_{21}, g_{22}, ..., g_{m1}, g_{m2}, ..., g_{mn}) = C_2 \tag{38}$$

$$... \quad ... \quad ...$$

$$f_n(g_{11}, g_{12}, ...g_{1n}, g_{21}, g_{22}, ..., g_{m1}, g_{m2}, ..., g_{mn}) = C_n \tag{39}$$

which can be easily solved. From this solution, the controller matrix K thus obtained locates the poles of the closed-loop system in the desired region. The effectiveness of the proposed methodologies can be conveniently presented by a numerical example in section three.

3. Design of robust controllers by neuro-fuzzy system

In the first section, the parametric state feedback matrix K for pole placement in a prescribed region was calculated by the following equation:

$$K = -B_0^{-1}(G_0 - G_\lambda)T^{-1} \tag{40}$$

where

$$G_\lambda = \begin{bmatrix} g_{11} & g_{12} & \cdots & g_{1n} \\ g_{21} & g_{22} & \cdots & g_{2n} \\ . & . & \cdots & . \\ g_{m1} & g_{m2} & \cdots & g_{mn} \end{bmatrix} \tag{41}$$

According to the previous section, g_{ij} is related to ω for all values of parameter $\omega \in (0,1)$. In other words

$$g_{ij} = f_{ij}(\omega) \quad \textit{for all } \omega \in (0,1), \; i = 1, 2, ., ., ., m \textit{ and } \; j = 1, 2, ., ., ., n \tag{42}$$

In this section, by using the search method based on the neuro-fuzzy hybrid system on the parameter ω in $(0,1)$, the parametric state feedback matrix K, and the parameters $g_{ij} = f_{ij}(\omega)$ are found such that the eigenvalue spectrum, $\Lambda = \{\lambda_1\lambda_2, ..., \lambda_n\}$ of the closed-loop matrix $A+BK$, is assigned to the closed-loop system with least sensitivity. Then by implementing an iterative method, a measure of conditioning of the eigenvector matrix X, corresponding to Λ, is minimized. Finally the robust controller in prescribed region is obtained. First, the required neuro-fuzzy hybrid system is described and then the iterative method for finding the eigenvector matrix X and the robust controller K is explained.

3.1. **Neuro-fuzzy hybrid system modeling.** The process of design a fuzzy inference system, is called fuzzy modeling. The fuzzy modeling has the following features.

a. According to the rule structure of a fuzzy inference system, for modeling a system by human expertise. The fuzzy modeling take the information that might not be easily employed in the other modeling approaches.

b. Conventional system identification techniques for designing the fuzzy modeling are based on the input-output data of the system.Thus the structure determination and parameters identification are usually two essential parts of fuzzy modeling system process.

• The structure determination, is the process to determine relevant input, number of MF's for each input, number of rules and types of fuzzy models.

• The parameter identification, is the process of identifying the values of parameters that can present the best performance of the model. In particular, the neuro-fuzzy modeling refers to the process of applying various learning algorithms, in the neural network technique, to design the adaptive fuzzy inference system. Consider the fuzzy rule base with m rules in the following form [16]-[19],

$$(43)\quad Rule\ k: \quad IF x_1 \ \ is\ A_1^k\ and\ x_2\ is\ A_2^k\ and\ ...\ and\ x_n\ is\ A_n^k\ THEN\ y\ is\ B^k$$

where x_i, $i = 1, 2, ..., n$ and y are the input and output variables, respectively; and A_i^k and B^k, $k = 1, 2, ...l$ are the input and output fuzzy sets that characterized by corresponding fuzzy membership functions. The neuro-fuzzy system with n input and m rules is in the following form

$$(44)\qquad y(x, \overline{x}, \delta, w) = w^T \mu(x, \overline{x}, \delta)$$

where

$\overline{x} = [\overline{x}_1^1, ..., \overline{x}_n^1, ..., \overline{x}_1^m, ..., \overline{x}_n^m]^T$, $\quad \delta = [\delta_1^1, ..., \delta_n^1, ..., \delta_1^m, ..., \delta_n^m]^T$,

$x = [x_1, x_2, ..., x_n]^T$, $\ w = [w_1, w_2, ..., w_m]^T\ \ and\ \ \mu = [\mu_1, \mu_2, ..., \mu_m]^T$.

such that $\overline{x}_i^k,\ \delta_i^k,\ \ w_k\ \ and\ \ \mu_k(x) = \frac{\Pi_{i=1}^n \mu_{A_i^k}(x)}{\sum_{k=1}^m \Pi_{i=1}^n \mu_{A_i^k}(x)}$ are the center, width of the activation function, the inference weight and value of output membership function, respectively. If $\overline{x}^*$, δ^* and w^* are the optimal value of $\overline{x}$, δ and w then by the universal approximation theorem, there exist a neuro-fuzzy system that can approximate uniformly the optimal values [15], [19]. If the algorithm used for training the neural network is combine of to algorithms, then the resulting system is called a hybrid system. Now, the neuro-fuzzy hybrid system, made up of the identifier system and universal approximator is introduced. First, the functional representation of the fuzzy system will be derived, and then the hybrid neuro-fuzzy system will be described. Fuzzy system and ANN system are both a universal approximator [19].

Consider the following fuzzy rule base:

$$If\ \omega\ is\ A^{\tau}\ Then\ J = F^{\tau}(\omega) \tag{45}$$

where A^{τ} is a fuzzy set in $[0,1]$ and ω is the parameter described in the first section and

$$F^{\tau}(\omega) \ = \frac{\|C_{\omega}^{\tau}\|_2}{\sqrt{n}} \tag{46}$$

such that $C_{\omega}^{\tau} = [c_{\omega}^{\tau}(j)]_{n\times 1}$ for $j = 1,2,...,n$ is the vector whose elements are the condition number, corresponding to value of $\omega \in [0,1]$ and

$$c_{\omega}^{\tau}(j) = \frac{1}{(y_{\omega}^{\tau}(j))x_{\omega}^{\tau}(j)} \tag{47}$$

where, $x_{\omega}^{\tau}(j)$ and $y_{\omega}^{\tau}(j)$ are respectively right and left eigenvector corresponding to $\lambda_{\omega}^{\tau}(j)$ in Λ_{ω}. Therefore

$$F^{\tau}(\omega) \ = \frac{\|C_{\omega}^{\tau}\|_2}{\sqrt{n}} = (\sum_{j=1}^{n} \frac{(c_{\omega}^{\tau}(j))^2}{n})^{\frac{1}{2}} \tag{48}$$

The fuzzy logic system with center average defuzzifier, max-product inference, singleton fuzzifier is of the following form [19]:

$$f(\omega) = \frac{\sum_{\tau=1}^{k} F^{\tau}(\omega)\mu_{A^{\tau}}(\omega)}{\sum_{\tau=1}^{k} \mu_{A^{\tau}}(\omega)} \tag{49}$$

By choosing a certain type of membership function such as a Gaussian membership function we can drive the fuzzy system in a functional form:

$$f(\omega) = \frac{\sum_{\tau=1}^{k} F^{\tau}(\omega)\exp(-(\frac{\omega-\overline{\omega}^{\tau}}{\sigma^{\tau}})^2)}{\sum_{\tau=1}^{k} \exp(-(\frac{\omega-\overline{\omega}^{\tau}}{\sigma^{\tau}})^2)} \tag{50}$$

By substituting, from (48) into (50) this functional takes the following form:

$$f(\omega) = \frac{\sum_{\tau=1}^{k}(\sum_{j=1}^{n} \frac{(c_{\omega}^{\tau}(j))^2}{n})^{\frac{1}{2}}\exp(-(\frac{\omega-\overline{\omega}^{\tau}}{\sigma^{\tau}})^2)}{\sum_{\tau=1}^{k} \exp(-(\frac{\omega-\overline{\omega}^{\tau}}{\sigma^{\tau}})^2)} \tag{51}$$

The functional of the form (51) is a universal approximator of condition number function J, over $[0,1]$. Any degree of accuracy can now be reached if $\overline{\omega}^{\tau}$ and σ^{τ}, are chosen properly. Universal approximator in (51) justifies the fuzzy logic system to our search problem. Here, the training fuzzy system using neural network learning method, which is a combination of least-squares estimation with back propagation is implemented. This learning method is supported by MATLAB software in the fuzzy logic toolbox.

Fuzzy systems can be trained by using various training algorithms based on the numerical data or training data. Assume that, A^{τ} is a fuzzy set with the membership function $\mu - A^{\tau}$ and (ω, F^{ω}) is given as the training data, the goal of the neuro-fuzzy hybrid system is to find values of the parameters $\overline{\varpi}^\tau$ and σ^τ, so that $f(\omega)$ is universal approximator of

$$J(\omega) \ = \frac{\|C_\omega^\tau\|_2}{\sqrt{n}} \tag{52}$$

In other words:

$$\sup_{\omega\in[0,1]} \|J(\omega) - f(\omega)\|_2 < tol \tag{53}$$

Since we can train the fuzzy system using hybrid method, we can call such a fuzzy system a fuzzy identifier. By this identifier system, we can find value(s) of ω such that the corresponding spectrum(s)of eigenvalues in a prescribed region, has the least sensitivity. Now, the eigenvectors matrix X^0, for use as initial matrix in the iterative method for calculating the robust controller, can be obtained.

3.2. **Iterative method for finding robust controller.** The object here is to choose eigenvectors $x_j,\ for\ j = 1, 2, ..., n$, corresponding to the spectrum Λ, such that each vector is as orthogonal as possible to the space spanned by remaining vectors; that is the angle between vector x_j and space, $\chi_j =< x_i, i \neq j >$, is maximized for all j. Equivalently we choose x_j to minimize the angle between x_j and the normalized vector y_j orthogonal to space χ_j. The solution is found here by an iteration method in which a matrix $X = [x_j],\ for\ j = 1, 2, ..., n$ at each step. The objective of the update is now to select a new vector x_j, for each $j = 1, 2, ..., n$ in order to minimize the measure of conditioning $\nu = \|X^{-1}\|_F^2$ over all x_j at each step. Thus at each step, a non-linearly constraint least-squares problem must be solved. This can be accomplished by QR decomposition method. Any set of independent vectors x_j may be used to start the procedure(See Kautsky,et.al. for details [12]). But in this paper, to procurability convergence in least steps, we choose x_j^0 the $j-th$ column of the eigenvector matrix X^0, (the initial eigenvector matrix). This eigenvector is found by the method of parameterizations and neuro-fuzzy system. The iteration is stopped when the reduction in measure ν is less than a given tolerance after a full sweep.

After finding the eigenvector matrix X we can calculate the state feedback matrix K. Let $\Gamma = A + BK$, be the closed-loop matrix and X the eigenvector matrix corresponding to Λ, then:

$$\Gamma X = X\Lambda \tag{54}$$

But the matrix X is non-defective. Therefore, the matrix $\Gamma = X\Lambda X^{-1}$, is constructed by solving the equation $X^T\Gamma^T = (X\Lambda)^T$ for Γ^T using a direct LU decomposition method. This process is stable for a well condition matrix X. To calculate matrix K, by QR decomposition we have:

$$B = \begin{bmatrix} Q_0 & Q_1 \end{bmatrix} \begin{bmatrix} R \\ 0 \end{bmatrix} \tag{55}$$

where R is an upper triangular matrix, thus [12] :

$$K = R^{-1} Q_0^T (\Gamma - A) \tag{56}$$

In the next subsection a numerical example is presented to illustrate the advantages of this new method for the design of optimal robust controller in a specified region.

4. ILLUSTRATIVE EXAMPLES

i) For pole placement in a prescribed region, the same continuous time problem by Kautsky, et.al. [12] is considered here.

$$A = \begin{bmatrix} 1.3800 & -0.2077 & 6.7150 & -5.6760 \\ -0.5814 & -4.2900 & 0 & 0.6750 \\ 1.0670 & 4.2730 & -6.6540 & 5.8930 \\ 0.048 & 4.273 & 1.3430 & -2.1040 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 \\ 5.6790 & 0 \\ 1.1360 & -3.1460 \\ 1.1360 & 0 \end{bmatrix}$$

It is desired to obtain a state feedback controller, which locates the poles of the closed-loop system in the region defined by $\beta \le real(lambda_i \le \alpha$. The transformed pair $(\tilde{B}, \tilde{A})$, in vector companion form and the corresponding transformation matrix are:

$$\tilde{A} = \begin{bmatrix} -5.2588 & 0.2498 & -1.2493 & 2.6983 \\ -1.3832 & -6.4092 & -11.0617 & 19.5403 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \tilde{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$T^{-1} = \begin{bmatrix} -0.004 & 0.1839 & 0 & -0.0393 \\ -0.0653 & 0.0098 & -0.3179 & 0.2687 \\ -0.0071 & -0.0071 & 0 & 0.0356 \\ -0.0473 & 0 & 0 & 0 \end{bmatrix}$$

clearly

$$B_0^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

and

$$G_0 = \begin{bmatrix} -5.2588 & 0.2498 & -1.2493 & 2.6983 \\ -1.3832 & -6.4092 & -11.0617 & 19.5403 \end{bmatrix}$$

Now let us consider

$$\tilde{A}_\lambda = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

with the same canonical structure as $\tilde{A}$. Here

$$\tilde{G}_\lambda = \begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \end{bmatrix}$$

is the parametric controller matrix in transformed space. Our aim is to obtain the set of equations relating these parameters such that the poles of A_λ are in the region defined by lines $x = \alpha = -1$ $x = \beta = -3$. Now it can be easily verified that

$$\det(\tilde{A}_\lambda - \lambda I) = \lambda^4 - (g_{11} + g_{22})\lambda^3 + (g_{11}g_{22} - g_{12}g_{21} - g_{13} - g_{24})\lambda^2 + (g_{22}g_{13} - g_{12}g_{23} + g_{11}g_{24} - g_{14}g_{21})\lambda + (g_{13}g_{24} - g_{14}g_{23}) \tag{57}$$

Now by using equations (26) - (33) we obtain values for the lower and upper bounds of c_i , $(for\ i = 1, 2, 3, 4)$ as follows:

$$t_{11} = 4 \quad t_{12} = 12 \tag{58}$$

$$t_{21} = 6 \quad t_{22} = 54 \tag{59}$$

$$t_{31} = 4 \quad t_{32} = 108 \tag{60}$$

$$t_{41} = 1 \quad t_{42} = 81 \tag{61}$$

Now from the characteristic equation of the closed-loop system in the transformed space we obtain the set of non-linear system of inequalities:

$$t_{11} \leq -(g_{11} + g_{22}) \leq t_{12} \tag{62}$$

$$t_{21} \leq (g_{11}g_{22} - g_{12}g_{21} - g_{13} - g_{24}) \leq t_{22} \tag{63}$$

$$t_{31} \leq (g_{22}g_{13} - g_{12}g_{23} + g_{11}g_{24} - g_{14}g_{21}) \leq t_{32} \tag{64}$$

$$t_{41} \leq (g_{13}g_{24} - g_{14}g_{23}) \leq t_{42} \tag{65}$$

By choosing $\omega = 0.75$, say and assuming $g_{21} = g_{22} = g_{23} = g_{24} = 1$ as was proposed, and substituting these in the above inequalities it follows that

$$g_{11} = -\omega t_{12} - (1 - \omega)t_{11} - 1 = -11 \tag{66}$$

$$-g_{12} - g_{13} = \omega t_{22} + (1-\omega)t_{21} + 1 - g_{11} = 54 \tag{67}$$

$$g_{13} - g_{12} - g_{14} = \omega t_{32} + (1-\omega)t_{31} - g_{11} = 93 \tag{68}$$

$$g_{13} - g_{14} = \omega t_{42} + (1-\omega)t_{41} = 61 \tag{69}$$

Here we have four equations with four unknowns, which can be solved readily. The solution yields

$$g_{11} = -11\ ,\ g_{12} = -32\ ,\ g_{13} = -22\ ,\ g_{14} = -83.$$

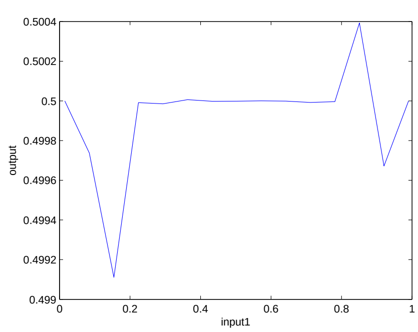

Figure.2 Universal approximator

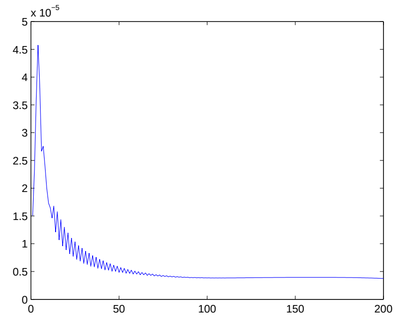

Figure.3 Performance error

Therefore

$$G_\lambda = \begin{bmatrix} -11 & -32 & -22 & -83 \\ 1 & 1 & 1 & 1 \end{bmatrix} \tag{70}$$

Producing

$$K = \begin{bmatrix} 0.0820 & -0.8736 & -0.2385 & 0.4319 \\ 0.9664 & -1.2970 & -0.6315 & 0.1455 \end{bmatrix} \tag{71}$$

It is a simple matter to check and see that the poles of the closed-loop system $A + BK$ now are $-2.8406 \pm 2.1373i$ and $-2.1594 \pm 0.4052i$, which clearly lie inside the specified region as required. However, the measure of robustness of this feedback matrix is not determined.

ii) For finding the optimal robust controller in the same prescribed region as in the above example, the method described in the section 2 is applied. Now by using the neuro-fuzzy hybrid system we have the following table:

Table.1: neuro-fuzzy system properties

In-MF	Number MF	Out-MF	Tol	Epoch	Err	ω
Gaussian	15	Linear	10^{-5}	200	1.174×10^{-6}	0.189

Figure 2 shows the variation of condition number with ω (from which the the minimum agrees with 0.189). Figure 3 shows the performance error with the epoch number of training. This error agrees with 1.174×10^{-6} after 200 epoches, and this is less than *tol* (10^{-5}). Therefore this approximator is a universal approximator.
The optimal eigenvector is:

$$X = \begin{bmatrix} -0.7807 & -0.7807 & 0.0546 - 0.1405i & 0.0546 + 0.1405i \\ -0.1809 + 0.1093i & -0.1809 - 0.1093i & -0.1337 + 0.4639i & -0.1337 - 0.4639i \\ 0.3056 - 0.4802i & 0.3056 + 0.4802i & 0.5724 + 0.1237i & 0.5724 - 0.1237i \\ -0.0712 + 0.1297i & -0.0712 - 0.1297i & 0.6335 & 0.6335 \end{bmatrix}$$

whose corresponding optimal robust controller matrix is:

$$K = \begin{bmatrix} 0.3865 & 0.1935 & 0.1196 & -0.7248 \\ 2.0172 & 0.4150 & -0.8368 & 1.0817 \end{bmatrix}$$

with the eigenvalue spectrum:
$\Lambda = \{-1.8143{+}5.1027i, -1.8143{-}5.1027i, -2.4976{+}3.4712i, -2.4976{-}3.4712i\}$

The measures of robustness in the specified region are presented in the following table :

Table.2: measure of robustness

$\|c\|_\infty$	$\kappa_2(X)$	$\|c\|_2$	$\|K\|_2$	Number of sweeps
1	1.3887	2	2.4722	4

Therefore, this controller matrix not only assigns the poles in the prescribed region but also is the most robust controller with minimum condition number 1.3887 for the eigenvector matrix of the closed-loop system.

Conclusion

In this work the explicit method based on the parametrization of feedback matrix for pole placement and the search method by implementing the neuro-fuzzy system to determine an optimal robust controller, or well-conditioned controller is presented. Specially the poles are located in a prescribed region such that least sensitivity to perturbations in the closed-loop matrix is achieved. For such robust solution it is shown that the criterions of robustness and the norm of feedback matrix are both optimized.
The idea for finding the robust controller suggested by Kautsky et.al [12] is extended. However, in our iterative method, the initial eigenvector, is the eigenvector that is found by the method of parameterizations and neuro-fuzzy system. This strategy guaranties convergence and minimizes the number of sweeps of iterations. The test example confirms better result and is numerically more efficient than the previous methods.

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