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Neutrosophic b-Locally Open Sets in Neutrosophic Topological Spaces

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Abstract. The main aim of this article is to introduce the notion of neutrosophic locally open set, neutrosophic locally closed set, neutrosophic b-locally open set, neutrosophic b-locally closed set, NLO*-set, NLC*-set, NLO**-set, NLO**-set, N-bLO**-set and N-bLC**-set via neutrosophic topological spaces, and investigate several properties of these classes of sets. Besides, we formulate several interesting theorems, propositions, remarks, etc. on neutrosophic topological spaces. Further, we furnish few illustrative examples on these classes of sets.

Keywords: Neutrosophic set, Neutrosophic topology, Neutrosophic locally open set, Neutrosophic b-locally open set.

2020 Mathematics subject classification: 54A40, 03E72, 03E72.

1. Introduction

Zadeh [40] grounded the notion of fuzzy set (in short FS) theory to deal with events having uncertainty. Later on, Atanassov [3] presented the concept of intuitionistic fuzzy set (in short IFS) theory by extending the notion of FS theory. Many times, the uncertainty events will have an indeterminacy part, which cannot be expressed using the concepts of crisp set, fuzzy set and intuitionistic fuzzy set. Keep in mind the importance of indeterminacy, Smarandache

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[32] introduced the concept of neutrosophic set (in short NS) as a generalization of IFS, where every element will have the degree of truth-membership, indeterminacy-membership and false-membership. Till now, many researcher around the globe applied the notion of NS and its extensions in their theoratical [6, 7, 11, 12, 15, 16, 17, 19] as well as practical research [8, 13, 14, 23, 28, 31, 34]. The notion of topological space have been generalized in many ways. Different types of open sets have been introduced and their properties have been investigated by many researcher. The generalization of the open sets has attracted many researchers to study different properties of the topological spaces. Chang [4] grounded the notions of fuzzy topological spaces. Afterwards, intuitionistic fuzzy topological space was introduced by Coker [5]. The concept of neutrosophic topological space (in short NTS) was grounded by Salama and Alblowi [29] in the year 2012. Salama and Alblowi [30] also studied the concept of generalized NS and generalized NTSs. The notion of neutrosophic semi open sets and neutrosophic semi closed sets was presented by Iswaraya and Bageerathi [21]. Arokiarani et al. [2] introduced several notations and functions via NTSs. Rao and Srinivasa [27] grounded the idea of neutrosophic pre-open set and neutrosophic pre-closed set. In the year 1996, Andrijevic [1] defined the concept of b-open set in topological space. Later on, Ebenanjar et al. [20] presented the concept of neutrosophic b-open sets via NTSs. In the year 2020, Das et al. [8] proposed a decision making approach under neutrosophic environment. Das and Pramanik [9] studied the generalized neutrosophic b-open set in NTSs. Das and Pramanik [10] also introduced the neutrosophic ϕ-open set and neutrosophic ϕ-continuous functions via NTSs. The notion of neutrosophic simply b-open set via NTSs was introduced by Das and Tripathy [18]. In 2021, Vadivel and Thangaraja [38] grounded the notion of e-continuous and somewhat e-continuity in N_{nc}-Topological Spaces. Later on, Vadivel and Thangaraja [39] further studied the notion of e-open sets via N_{nc}-Topological Spaces. Afterwards, Sudha et al. [33] established the notion of separation axioms via N_{nc}-Topological Spaces. The notions of b-locally open set was first studied by Nasef [24] in 2001. In 1963, Kelly [22] first grounded the concept of bitopological space. Thereafter, the idea of b-locally closed sets via bitopological spaces was established by Rajesh [26]. Later on, the notion of b-locally open sets via bitopological space was presented by Tripathy and Sarma [36]. Tripathy and Sarma [37] also studied the notion of pairwise b-locally open sets and pairwise neutrosophic closed sets in bitopological space. Afterwards, Tripathy and Debnath [35] introduced the fuzzy b-locally open sets via fuzzy bitopological space. Recently, Ozturk and Ozkan [25] grounded the notion of neutrosophic bitopological spaces.

The main focus of this article is to introduce the notion of neutrosophic locally open set, neutrosophic locally closed set, neutrosophic b-locally open set, neutrosophic b-locally closed set, $NbLO^*$-set, $NBLC^*$-set, $NbLO^{**}$-set, $NbLC^{**}$-set, $NbLO^{***}$-set and $NbLC^{***}$-set via NTSs, and formulate several interesting results on them.

2. Some relevant results

In this section, we provide few basic definitions and results those will be used throughout. Also, some examples have been provided to clarify the situations.

Definition 2.1. [32] Let X be an universal set. An NS A over X is a set contains triplet having truth, indeterminacy and false membership values denoted by T_A, I_A, F_A in the interval [0, 1]. The NS A is denoted as follows: $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X$, and $T_A(x), I_A(x), F_A(x) \in [0, 1]\}$, where $0 \leq T_A(x) + F_A(x) + I_A(x) \leq 3$, for all $x \in X$.
The null NS (0_N) and the whole NS (1_N) over X are defined as follows:

(1) $0_N = \{(x, 0, 0, 1) : x \in X\}$;
(2) $1_N = \{(x, 1, 0, 0) : x \in X\}$.

Clearly, $0_N \subseteq 1_N$.

Remark 2.2. [32] For any NS A over a fixed set X, the followings hold:

(1) $0_N \subseteq A$
(2) $A \subseteq 1_N$

Definition 2.3. [32] Let $A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ be a NS over X. Then, the complement of A is defined by A^c =$\{(x, 1-T_A(x), 1-I_A(x), 1-F_A(x)) : x \in X\}$.

Definition 2.4. [32] An NS A=$\{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ is contained in the other NS B=$\{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}$ (i.e., $A \subseteq B$) if and only if $T_A(x) \leq T_B(x), I_A(x) \geq I_B(x), F_A(x) \geq F_B(x)$, for each $x \in X$.

Definition 2.5. [32] Suppose that A=$\{(x, T_A(x), I_A(x), F_A(x)) : x \in X\}$ and B=$\{(x, T_B(x), I_B(x), F_B(x)) : x \in X\}$ be any two NSs over X. Then, the union and intersection of A and B i.e., $A \cup B$ and $A \cap B$ is defined as follows:

(1) $A \cup B$=$\{(x, T_A(x) \vee T_B(x), F_A(x) \wedge F_B(x), I_A(x) \wedge I_B(x)) : x \in X\}$;
(2) $A \cap B$=$\{(x, T_A(x) \wedge T_B(x), F_A(x) \vee F_B(x), I_A(x) \vee I_B(x)) : x \in X\}$.

The notion of neutrosophic topological spaces is defined as follows:

Definition 2.6. [29] Let X be a non-empty set and τ be the collection of neutrosophic sets defined over X. Then, τ is said to be an neutrosophic topology (in short NT) on X if the following conditions hold:

(1) 0_N, $1_N \in \tau$;
(2) U_1, $U_2 \in \tau \implies U_1 \cup U_2 \in \tau$;
(3) $\cup\, U_i \in \tau$ whenever $\{U_i\colon\ i \in \Delta\} \subseteq \tau$.

Then, the pair (X,τ) is called an neutrosophic topological space (in short NTS). The members of τ are called neutrosophic open set (in short NOS). An NS D is called an neutrosophic closed set (in short NCS) if and only if D^c is an NOS.

EXAMPLE 2.7. Let X=$\{a,b\}$ be a fixed set. Suppose that P=$\{(a,0.5,0.5,0.4), (b,0.6,0.7,0.4)$: $a,b \in X\}$, Q=$\{(a,0.4,0.6,0.8), (b,0.4,0.9,0.8)$: $a,b \in X\}$, and R=$\{(a,0.5,0.6,0.4), (b,0.4,0.8,0.5)$: $a,b \in X\}$ be three neutrosophic sets over X. Then, the family τ =$\{0_N, 1_N, P, Q, R\}$ is a neutrosophic topology on X.

EXAMPLE 2.8. Let X=$\{a,b,c\}$. Suppose that A=$\{(a,0.5,0.5,0.4), (b,0.6,0.7,0.4), (c,0.5,0.6,0.8)$: a, b, $c \in X\}$, B=$\{(a,0.4,0.6,0.8), (b,0.4,0.9,0.8), (c,0.3,0.7,0.8)$: a, b, $c \in X\}$, and C=$\{(a,0.3,0.5,0.8), (b,0.4,0.9,0.9), (c,0.2,0.8,0.9)$: a, b, $c \in X\}$ be three neutrosophic sets over X. Then, the collection τ=$\{0_N,1_N$, A, B, $C\}$ is not a neutrosophic topology on X, because $B \cap C \notin \tau$.

The neutrosophic interior and neutrosophic closure of a neutrosophic set is defined as follows:

Definition 2.9. [29] Let (X,τ) be an NTS and U be an NS in X. Then, the neutrosophic interior (in short N_{int}) and neutrosophic closure (in short N_{cl}) of U are defined as follows:

(1) $N_{int}(U) = \cup\{E : E$ is an NOS in X, and $E \subseteq U\}$;
(2) $N_{cl}(U) = \cap\{F : F$ is an NCS in X, and $U \subseteq F\}$.

Remark 2.10. [29] Clearly, $N_{int}(U)$ is the largest NOS in (X,τ) which is contained in U and $N_{cl}(U)$ is the smallest NCS in (X,τ) which contains U.

Proposition 2.11. [29] *For any neutrosophic sub-set B of an NTS (X,τ), we have the following:*

(1) $N_{int}(B^c) = (N_{cl}(B))^c$;
(2) $N_{cl}(B^c) = (N_{int}(B))^c$.

Proposition 2.12. [29] *Let (X,τ) be an NTS. Suppose that A and B be any two NSs over X. Then, the following properties hold:*

(1) $A \subseteq N_{cl}(A)$;
(2) $N_{int}(A) \subseteq A$;
(3) $N_{int}(A) \subseteq N_{cl}(A)$;
(4) $A \subseteq B \implies N_{int}(A) \subseteq N_{int}(B)$;
(5) $A \subseteq B \implies N_{cl}(A) \subseteq N_{cl}(B)$;
(6) $N_{cl}(1_N) = 1_N$;
(7) $N_{cl}(0_N) = 0_N$;
(8) $N_{int}(1_N) = 1_N$;
(9) $N_{int}(0_N) = 0_N$;
(10) $N_{cl}(A \cup B) = N_{cl}(A) \cup N_{cl}(B)$;

(11) $N_{int}(A \cup B) \supseteq N_{int}(A) \cup N_{int}(B)$;
(12) $N_{int}(A \cap B) = N_{int}(A) \cap N_{int}(B)$;
(13) $N_{cl}(A \cap B) \subseteq N_{cl}(A) \cap N_{cl}(B)$;
(14) *G is a neutrosophic closed set if and only if* $N_{cl}(G) = G$;
(15) *G is a neutrosophic open set if and only if* $N_{int}(G) = G$.

Definition 2.13. Let (X, τ) be an NTS, and G be an NS over X. Then, G is called an

(1) [21] neutrosophic semi-open (in short NSO) set if and only if $G \subseteq N_{cl}(N_{int}(G))$;
(2) [27] neutrosophic pre-open (in short NPO) set if and only if $G \subseteq N_{int}(N_{cl}(G))$.

Remark 2.14. [27] Every neutrosophic open set is an neutrosophic pre-open set.

Remark 2.15. [21] Every neutrosophic open set is an neutrosophic semi-open set.

Definition 2.16. [20] In an NTS (X, τ), an NS G is said to be an

(1) neutrosophic b-open set iff $G \subseteq N_{int}(N_{cl}(G)) \cup N_{cl}(N_{int}(G))$;
(2) neutrosophic b-closed set iff $N_{int}(N_{cl}(G)) \cup N_{cl}(N_{int}(G)) \subseteq G$.

Remark 2.17. [20] An NS G is said to be an neutrosophic b-closed set in (X, τ) if and only if its complement is an neutrosophic b-open set in (X, τ).

Theorem 2.18. [20] *Let (X, τ) be an NTS. Then, every neutrosophic pre-open (neutrosophic semi-open) set is also an neutrosophic b-open set.*

Remark 2.19. The converse of the above theorem may not be true in general. This follows from the following example.

Example 2.20. Let X=$\{a, b\}$ and τ =$\{0_N, 1_N, \{(a,0.3,0.5,0.6), (b, 0.4, 0.6, 0.7) : a, b \in X\}, \{(a,0.5,0.2,0.4), (b, 0.6, 0.3, 0.5) : a, b \in X\}\}$ be an neutrosophic topology on X. Then, A=$\{(a,0.6,0.5,0.6), (b, 0.8, 0.6, 0.7) : a, b \in X\}$ is an neutrosophic b-open set in (X, τ). But A is not an neutrosophic pre-open set in (X, τ).
Similarly, the NS B=$\{(a, 0.6, 0.1, 0), (b, 0.7, 0.5, 0) : a, b \in X\}$ is an neutrosophic b-open set in (X, τ). But B is not an neutrosophic semi-open set in (X, τ).

Theorem 2.21. *In an NTS (X, τ), the union of two neutrosophic b-open sets is again a neutrosophic b-open set.*

Proof. Let A and B be any two neutrosophic b-open sets in an NTS (X, τ). Therefore,

$$A \subseteq N_{int}(N_{cl}(A)) \cup N_{cl}(N_{int}(A)) \tag{2.1}$$

and

$$B \subseteq N_{int}(N_{cl}(B)) \cup N_{cl}(N_{int}(B)) \tag{2.2}$$

For any two NSs A and B, it is known that $A \subseteq A \cup B$ and $B \subseteq A \cup B$. Now,

$$A \subseteq A \cup B \implies N_{cl}(N_{int}(A)) \subseteq N_{cl}(N_{int}(A \cup B)) \tag{2.3}$$

$$A \subseteq A \cup B \implies N_{int}(N_{cl}(A)) \subseteq N_{int}(N_{cl}(A \cup B)) \tag{2.4}$$

$$B \subseteq A \cup B \implies N_{cl}(N_{int}(B)) \subseteq N_{cl}(N_{int}(A \cup B)) \tag{2.5}$$

and

$$B \subseteq A \cup B \implies N_{int}(N_{cl}(B)) \subseteq N_{int}(N_{cl}(A \cup B)) \tag{2.6}$$

From eq. (2.1) and eq. (2.2) we have,
$A \cup B$
$\subseteq N_{cl}(N_{int}(A)) \cup N_{int}(N_{cl}(A)) \cup N_{cl}(N_{int}(B)) \cup N_{int}(N_{cl}(B))$
$\subseteq N_{cl}(N_{int}(A{\cup}B)){\cup}N_{int}(N_{cl}(A{\cup}B)){\cup}N_{cl}(N_{int}(A{\cup}B)){\cup}N_{int}(N_{cl}(A{\cup}B))$ [by eqs. (2.3), (2.4), (2.5), (2.6)]
$=N_{cl}(N_{int}(A \cup B)) \cup N_{int}(N_{cl}(A \cup B))$
$\implies A \cup B \subseteq N_{cl}(N_{int}(A \cup B)) \cup N_{int}(N_{cl}(A \cup B))$.
Therefore, $A \cup B$ is an neutrosophic b-open set in (X, τ). Hence, the union of two neutrosophic b-open sets is again an neutrosophic b-open set in (X, τ). □

Remark 2.22. The intersection of two neutrosophic b-open sets may not be an neutrosophic b-open set in general. This follows from the following example.

EXAMPLE 2.23. Let X=$\{a, b\}$ be a fixed set. Then, τ=$\{0_N, 1_N, \{(a$,0.3,0.5,0.6), $(b, 0.4, 0.4, 0.5) : a, b \in X\}$, $\{(a$,0.5,0.2,0.4), $(b, 0.6, 0.1, 0.3) : a, b \in X\}\}$ is an neutrosophic topology on X. Clearly, H=$\{(a$,0.4,0.5,0.4), $(b$,0.5,0.4,0.3): $a, b \in X\}$, K=$\{(a, 0, 1, 0), (b$,0.1,0.9,0.9): $a, b \in X\}$ are two neutrosophic b-open sets in (X, τ). But, their intersection $H{\cap}K$=$\{(a$,0,1,0.4), $(b, 0.1, 0.9, 0.9) : a, b \in X\}$ is not an neutrosophic b-open set in (X, τ).

Theorem 2.24. *The intersection of two neutrosophic b-closed sets in an NTS* (X, τ) *is again an neutrosophic b-closed set in* (X, τ).

Proof. Let A and B be any two neutrosophic b-closed sets in an NTS (X, τ). Since, the complement of an neutrosophic b-closed set is an neutrosophic b-open set, so A^c and B^c are neutrosophic b-open sets in (X, τ). In the previous theorem, it is shown that the union of two neutrosophic b-open sets is again an neutrosophic b-open set in (X, τ). So $A^c \cup B^c$ is an neutrosophic b-open set in (X, τ). Now, $(A \cap B)^c$=$A^c \cup B^c$, which is an neutrosophic b-open set in (X, τ). Hence, its complement $A \cap B$ is an neutrosophic b-closed set in (X, τ). Therefore, the intersection of two neutrosophic b-closed sets in (X, τ) is again an neutrosophic b-closed set in (X, τ). □

Remark 2.25. The union of two neutrosophic b-closed sets may not be a neutrosophic b-closed set. This follows from the following example.

EXAMPLE 2.26. Let $X=\{a, b\}$. Then, $\tau=\{0_N, 1_N, \{(a,0.3,0.5,0.6), (b,0.4,0.4,0.5)$: $a, b \in X\}, \{(a,0.5,0.2,0.4), (b,0.6,0.1,0.3)$: $a, b \in X\}\}$ is an neutrosophic topology on X. It can be verified that $H=\{(a,0.6,0.5,0.6), (b,0.5,0.6,0.7)$: $a, b \in X\}$, $K=\{(a,1,0,1), (b,0.9,0.1,0.1)$: $a, b \in X\}$ are two neutrosophic b-closed sets, since their complements are neutrosophic b-open sets. But their union $H \cup K=\{(a,1,0,0.6), (b,0.9,0.1,0.1)$: $a, b \in X\}$ is not an neutrosophic b-closed set.

3. Neutrosophic b-locally open set

In this section, we introduce the concept of some open sets namely neutrosophic locally open set, neutrosophic b-locally open set, $NbLO^*$-set, $NbLO^{**}$-set, $NbLO^{***}$-set via NTSs, and formulate several interesting results on them.

Definition 3.1. Let (X, τ) be an NTS. An NS G over X is said to be an neutrosophic locally open (in short NLO) set if $G = H \cup K$, where H is an neutrosophic open set and K is an neutrosophic closed set in (X, τ). The complement of G i.e., G^c is called an neutrosophic locally closed (in short NLC) set.
Clearly, the null NS (0_N) and the whole NS (1_N) are both NLO set and NLC set in (X, τ).

Definition 3.2. In an NTS (X, τ), an NS G is said to be an neutrosophic b-locally open (in short $NbLO$) set if there exist an neutrosophic b-open set H and an neutrosophic b-closed set K in X such that $G = H \cup K$. The complement of G i.e., G^c is called an neutrosophic b-locally closed (in short $NbLC$) set.
Clearly, the null NS (0_N) and the whole NS (1_N) are both $NbLO$ set and $NbLC$ set in (X, τ).

Definition 3.3. In an NTS (X, τ), an NS G is said to be an $NbLO^*$ set if $G = H \cup K$, where H is an neutrosophic b-open set and K is an neutrosophic closed set in (X, τ). The complement of G i.e., G^c is called an $NbLC^*$ set.
Clearly, the null NS (0_N) and the whole NS (1_N) are both $NbLO^*$ set and $NbLC^*$ set in (X, τ).

Definition 3.4. In an NTS (X, τ), an NS G is said to be an $NbLO^{**}$ set if $G = H \cup K$, where H is an neutrosophic open set and K is an neutrosophic b-closed set in (X, τ). The complement of G i.e., G^c is called an $NbLC^{**}$ set.
Clearly, the null NS (0_N) and the whole NS (1_N) are both $NbLO^{**}$ set and $NbLC^{**}$ set in (X, τ).

Definition 3.5. In an NTS (X, τ), an NS G is said to be an $NbLO^{***}$ set if there exist an neutrosophic b-open set H in (X, τ) such that $G=H \cup H^c$.

Remark 3.6. The collection of all NLO, $NbLO$, $NbLO^*$, $NbLO^{**}$, $NbLO^{***}$ sets in an NTS (X, τ) may be denoted by $NLO(X)$, $NbLO(X)$, $NbLO^*(X)$, $NbLO^{**}(X)$, $NbLO^{***}(X)$.

Theorem 3.7. *Let (X,τ) be an NTS, and A be an NS over X. If $A \in NLO(X)$, then $A \in NbLO^*(X)$.*

Proof. Let $A \in NLO(X)$. Therefore, there exist an neutrosophic open set G and an neutrosophic closed set H such that $A{=}G{\cup}H$. Since, every neutrosophic open set is an neutrosophic b-open set, so G is an neutrosophic b-open set in (X,τ). Therefore, $A{=}G\cup H$, where G is an neutrosophic b-open set and H is an neutrosophic closed set in (X,τ). Hence, $A \in NbLO^*(X)$. □

Remark 3.8. The converse of the above theorem is not necessarily true. This follows from the following example.

Example 3.9. Let $X{=}\{a, b\}$. Then, the collection $\tau{=}\{0_N, 1_N, \{(a,0.8,0.4,0.9), (b,0.6,0.5,0.7)\colon a,b \in X\}, \{(a,0.7,0.5,1), (b,0.5,0.6,0.8)\colon a,b \in X\}, \{(a,0.6,0.6,1), (b,0.4,0.7,0.9)\colon a,b \in X\}\}$ is an neutrosophic topology on X. Let us consider an NS $A{=}\{(a,0.5,0.3,0), (b,0.7,0.2,0)\colon a,b \in X\}$ over X. Now, one can write $A{=}H \cup K$, where $H{=}\{(a,0.5,0.3,0), (b,0.7,0.2,0)\colon a,b \in X\}$ and $K{=}\{(a,0.2,0.6,0.1), (b,0.4,0.5,0.3)\colon a,b \in X\}$ are two NSs over X. Here, K is an neutrosophic closed set and H is an neutrosophic b-open set. Therefore, one can find an neutrosophic closed set K and an neutrosophic b-open set H such that $A{=}H{\cup}K$. Hence, $A \in NbLO^*(X)$. But, $A \notin NLO(X)$, because H is not an neutrosophic open set in the NTS (X,τ).

Theorem 3.10. *Let (X,τ) be an NTS, and A be an NS over X. If $A \in NLO(X)$, then $A \in NbLO^{**}(X)$.*

Proof. Let $A \in NLO(X)$. Therefore, there exist an neutrosophic open set G and an neutrosophic closed set H such that $A{=}G{\cup}H$. Since, every neutrosophic closed set is an neutrosophic b-closed set, so H is an neutrosophic b-closed set in (X,τ). Therefore, $A{=}G\cup H$, where G is an neutrosophic open set and H is an neutrosophic b-closed set in (X,τ). Hence, $A \in NbLO^{**}(X)$. □

Remark 3.11. The converse of the above theorem is not necessarily true. This follows from the following example.

Example 3.12. Let $X{=}\{a, b\}$. Then, $\tau{=}\{0_N, 1_N, \{(a,0.3,0.5,0.6), (b,0.4,0.4,0.5)\colon a,b \in X\}, \{(a,0.5,0.2,0.4), (b,0.6,0.1,0.3)\colon a,b \in X\}\}$ is an neutrosophic topology on X. Let us consider an NS $A{=}\{(a,0.6,0.2,0.4), (b,0.6,0.1,0.3)\colon a,b \in X\}$ over X. Now, we can write $A{=}K{\cup}H$, where $K{=}\{(a,0.5,0.2,0.4), (b,0.6,0.1,0.3)\colon a,b \in X\}$ and $H{=}\{(a,0.6,0.5,0.6), (b,0.5,0.6,0.7)\colon a,b \in X\}$ are two NSs over X. Here, K is an neutrosophic open set and H is an neutrosophic b-closed set (since H^c is an neutrosophic b-open set). Therefore, one can find an neutrosophic open set K and an neutrosophic b-closed set H such that $A{=}K \cup H$. Hence, $A \in NbLO^{**}(X)$. But, $A \notin NLO(X)$, because H is not an neutrosophic closed set in (X,τ).

Theorem 3.13. *Let A and B be any two NSs in (X, τ). If $A \in NLO(X)$ and B is an neutrosophic open set in (X, τ), then $A \cup B \in NLO(X)$.*

Proof. Let $A \in NLO(X)$. Therefore, there exists an neutrosophic open set G and an neutrosophic closed set H such that $A{=}G \cup H$. Also, given that B is an neutrosophic open set in (X, τ). Now, $A \cup B{=}(G \cup H) \cup B{=}(G \cup B) \cup H{=}K \cup H$, where $K{=}G \cup B$ is an neutrosophic open set (because the union of two neutrosophic open sets is again an neutrosophic open set) in (X, τ). Hence, $A \cup B \in NLO(X)$. □

Theorem 3.14. *Let (X, τ) be an NTS and A be an NS over X. If $A \in NbLO^*(X)$, then $A \in NbLO(X)$.*

Proof. Suppose that $A \in NbLO^*(X)$. Therefore, there exist an neutrosophic b-open set G and an neutrosophic closed set H such that $A{=}G{\cup}H$. Since, every neutrosophic closed set is an neutrosophic b-closed set, so H is an neutrosophic b-closed set in (X, τ). Therefore, $A{=}G \cup H$, where G is an neutrosophic b-open set and H is an neutrosophic b-closed set in the NTS (X, τ). Hence, $A \in NbLO(X)$. □

Remark 3.15. The converse of the above theorem is not necessarily true, which follows from the following example.

Example 3.16. Let $X{=}\{a, b\}$ and $\tau{=}\{0_N, 1_N, \{(a,0.3,0.5,0.6), (b,0.4,0.4,0.5){:}a, b \in X\}, \{(a,0.5,0.2,0.4), (b,0.6,0.1,0.3){:}a, b \in X\}\}$ be an neutrosophic topology on X. Let us consider an NS $A{=}\{(a,0.6,0.1,0), (b,0.7,0.4,0){:}a, b \in X\}$ over X. One can write $A{=}H \cup K$, where $H{=}\{(a,0.4,0.5,0.4), (b,0.5,0.4,0.3){:}a, b \in X\}$ and $K{=}\{(a,0.4,0.9,1), (b,0.3,0.5,1){:}a, b \in X\}$ are two NSs over X. Here, K is an neutrosophic b-closed set and H is an neutrosophic b-open set in (X, τ). Therefore, one can find an neutrosophic b-open set H and an neutrosophic b-closed set K in (X, τ) such that $A{=}H \cup K$. Hence, $A \in NbLO(X)$. But, $A \notin NbLO^*(X)$ because K is not an neutrosophic closed set in (X, τ).

Theorem 3.17. *Let (X, τ) be an NTS, and A be an NS over X. If $A \in NbLO^{**}(X)$, then $A \in NbLO(X)$.*

Proof. Suppose that $A \in NbLO^{**}(X)$. Therefore, there exist an neutrosophic open set G and an neutrosophic b-closed set H such that $A = G \cup H$. Since, every neutrosophic open set is an neutrosophic b-open set, so G is an neutrosophic b-open set. Therefore, $A = G \cup H$, where G is an neutrosophic b-open set and H is an neutrosophic b-closed set. Hence, $A \in NbLO(X)$. □

Remark 3.18. The converse of the above theorem is not necessarily true. It is clear from the following example.

Example 3.19. Let $X{=}\{a, b\}$ be a fixed set. Clearly, $\tau{=}\{0_N, 1_N, \{(a,0.8,0.4,0.9), (b,0.6,0.5,0.7){:}\ a, b \in X\}, \{(a,0.7,0.5,1), (b,0.5,0.6,0.8){:}\ a, b \in X\}, \{(a,0.6,0.6,1),$

(b,0.4,0.7,0.9): a, $b \in X$}} be an neutrosophic topology on X. Let us consider an NS A={(a,0.7,0.3,0), (b,0.7,0.2,0): a, $b \in X$} over X. One can write A=$H \cup K$, where H={(a,0.5,0.3,0), (b,0.7,0.2,0): a, $b \in X$} and K={(a,0.7,0.8,0.5), (b,0.4,0.5,0.3): a, $b \in X$} are two NSs over X. Here, K is an neutrosophic b-closed set and H is an neutrosophic b-open set in (X, τ). Therefore, one can find an neutrosophic b-open set H and an neutrosophic b-closed set K such that A=$H \cup K$. Hence, $A \in NbLO(X)$. But, $A \notin NbLO^{**}(X)$, because H is not an neutrosophic open set in (X, τ).

Theorem 3.20. *Let (X, τ) be an NTS, and A be an NS over X. If $A \in NbLO^{***}(X)$, then $A \in NbLO(X)$.*

Proof. Let $A \in NbLO^{***}(X)$. Therefore, there exist an neutrosophic b-open set H such that A=$H \cup H^c$. Since, H is an neutrosophic b-open set, so H^c is an neutrosophic b-closed set. Hence, A=$H \cup H^c$, where H is an neutrosophic b-open set and H^c is an neutrosophic b-closed set. Therefore, $A \in NbLO(X)$. □

Remark 3.21. The converse of the above theorem is not necessarily true, which follows from the following example.

EXAMPLE 3.22. Let X={a, b} be a fixed set. Then, the family τ={0_N, 1_N, {(a,0.6,0.6,1), (b,0.4,0.7,0.9): $a, b \in X$}, {(a,0.8,0.4,0.9), (b,0.6,0.5,0.7): $a, b \in X$}, {(a,0.7,0.5,1), (b,0.5,0.6,0.8): $a, b \in X$}} be an neutrosophic topology on X. Let us consider an NS A={(a,0.7,0.3,0), (b,0.7,0.2,0): $a, b \in X$} over X. Here, one can write A=$H \cup K$, where H={(a,0.7,0.8,0.5), (b,0.4,0.5,0.3): $a, b \in X$} and K={(a,0.5,0.3,0), (b,0.7,0.2,0): $a, b \in X$} are two NSs over X. Here, H is an neutrosophic b-closed set and K is an neutrosophic b-open set in the NTS (X, τ). Therefore, one can find an neutrosophic b-open set K and an neutrosophic b-closed set H such that A=$H \cup K$. Hence, $A \in NbLO(X)$. But, $A \notin NbLO^{***}(X)$, because H is not the complement of K.

Theorem 3.23. *Let A and B be any two NSs in (X, τ). If $A \in NbLO(X)$ and B is an neutrosophic b-open set, then $A \cup B \in NbLO(X)$.*

Proof. Let $A \in NbLO(X)$. Therefore, there exists an neutrosophic b-open set G and an neutrosophic b-closed set H such that A=$G \cup H$. Also, given that B is an neutrosophic b-open set in (X, τ). Now, $A \cup B$=$(G \cup H) \cup B$=$(G \cup B) \cup H$=$K \cup H$, where K=$G \cup B$ is an neutrosophic b-open set (because the union of two neutrosophic b-open sets is again an neutrosophic b-open set) in (X, τ). Hence, $A \cup B \in NbLO(X)$. □

Theorem 3.24. *Let A and B be any two NSs in an NTS (X, τ). If $A \in NbLO^*(X)$ and B is either neutrosophic closed set or neutrosophic open set in (X, τ), then $A \cup B \in NbLO^*(X)$.*

Proof. Let $A \in NbLO^*(X)$, and B is either neutrosophic open set or neutrosophic closed set in (X, τ). Then, there are two case.

Case-1: $A \in NbLO^*(X)$ and B is an neutrosophic open set in (X, τ).
Since, $A \in NbLO^*(X)$, so there exists an neutrosophic b-open set G and an neutrosophic closed set H such that A=$G \cup H$. It is known that, every neutrosophic open set is an neutrosophic b-open set, so B is an neutrosophic b-open set. Now, $A \cup B$=$(G \cup H) \cup B$=$(G \cup B) \cup H$=$M \cup H$, where M=$G \cup B$ is an neutrosophic b-open set (because the union of two neutrosophic b-open sets is again an neutrosophic b-open set) and H is an neutrosophic closed set in (X, τ). Hence, $A \cup B \in NbLO^*(X)$.
Case-2: $A \in NbLO^*(X)$ and B is an neutrosophic closed in (X, τ).
Since, $A \in NbLO^*(X)$, so there exists an neutrosophic b-open set G and an neutrosophic closed set H such that A=$G \cup H$. Now, $A \cup B$=$(G \cup H) \cup B$=$G \cup (H \cup B)$=$G \cup M$, where M=$H \cup B$ is an neutrosophic closed set (because the union of two neutrosophic closed sets is again an neutrosophic closed set) and G is an neutrosophic b-open set. Hence, $A \cup B \in NbLO^*(X)$. □

Theorem 3.25. *Let A and B be any two NSs in an NTS (X, τ). If $A \in NbLO^{**}(X)$ and B is an neutrosophic open set, then $A \cup B \in NbLO^{**}(X)$.*

Proof. Let $A \in NbLO^{**}(X)$ and B is an neutrosophic open set in (X, τ). Since, $A \in NbLO^{**}(X)$, so there exists an neutrosophic open set G and an neutrosophic b-closed set H such that A=$G \cup H$. Now, $A \cup B$=$(G \cup H) \cup B$=$(G \cup B) \cup H$=$M \cup H$, where M=$G \cup B$ is an neutrosophic open set (because the union of two neutrosophic open sets is again an neutrosophic open set) and H is an neutrosophic b-closed set in (X, τ). Hence, $A \cup B \in NbLO^{**}(X)$. □

Theorem 3.26. *Let A be an NS in an NTS (X, τ). Then, $A \in NbLO^{**}(X)$ if and only if A=$G \cup N_{int}(A)$ for some neutrosophic b-closed set G.*

Proof. Let $A \in NbLO^{**}(X)$. Therefore, there exists an neutrosophic open set H and an neutrosophic b-closed set G such that A=$H \cup G$. Now, $G \subseteq H \cup G$=A and $N_{int}(A) \subseteq A$. So,

$$G \cup N_{int}(A) \subseteq A \tag{3.1}$$

Further, we have $H \subseteq A$
$\implies N_{int}(H) \subseteq N_{int}(A)$
$\implies H = N_{int}(H) \subseteq N_{int}(A)$ [Since H is an neutrosophic open set]
$\implies H \subseteq N_{int}(A)$
$\implies G \cup H \subseteq G \cup N_{int}(A)$
$\implies A = H \cup G \subseteq G \cup N_{int}(A)$
Therefore,

$$A \subseteq G \cup N_{int}(A) \tag{3.2}$$

From eq. (3.1) and eq. (3.2), it is clear that $A = G \cup N_{int}(A)$.
Conversely, let $A = G \cup N_{int}(A)$ for some neutrosophic b-closed set. It is known that, $N_{int}(A)$ is an neutrosophic open set. Therefore, there exist an

neutrosophic b-closed set G and an neutrosophic open set $N_{int}(A)$ in (X, τ) such that $A = G \cup N_{int}(A)$. Hence, $A \in NbLO^{**}(X)$. □

Theorem 3.27. *If $A, B \in NLO(X)$, then $A \cup B \in NLO(X)$.*

Proof. Let $A, B \in NLO(X)$. Therefore, there exists two neutrosophic open sets G, H and two neutrosophic closed sets S, T such that $A = G \cup S$ and $B = H \cup T$. Now, $A \cup B = (G \cup S) \cup (H \cup T) = (G \cup H) \cup (S \cup T) = M \cup N$, where $M = G \cup H$ is an neutrosophic open set and $N = S \cup T$ is an neutrosophic closed set. Hence, $A \cup B \in NLO(X)$. □

Theorem 3.28. *If $A, B \in NbLO^*(X)$, then $A \cup B \in NbLO^*(X)$.*

Proof. Let $A, B \in NbLO^*(X)$. Therefore, there exists two neutrosophic b-open sets G, H and two neutrosophic closed sets S, T such that $A = G \cup S$ and $B = H \cup T$. Now, $A \cup B = (G \cup S) \cup (H \cup T) = (G \cup H) \cup (S \cup T) = M \cup N$, where $M = G \cup H$ is an neutrosophic b-open set (because the union of two neutrosophic b-open sets is again an neutrosophic b-open set) and $N = S \cup T$ is an neutrosophic closed set (because the union of two neutrosophic closed sets is again an neutrosophic closed set). Hence, $A \cup B \in NbLO^*(X)$. □

4. Conclusion

In this article, we have introduced the notion of neutrosophic locally open set, neutrosophic b-locally open set, $NbLO^*$-set, $NbLO^{**}$-set, $NbLO^{***}$-set via NTSs, and investigate some of their basic properties. By defining neutrosophic locally open set, neutrosophic b-locally open set, $NbLO^*$-set, $NbLO^{**}$-set, $NbLO^{***}$-set, we have formulated some interesting results on them via NTSs. Further, we have given few illustrative examples on them via neutrosophic topological spaces.

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