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Erratum " Some Result on Simple Hyper K- Algebras ", Iranian Journal of Mathematical Sciences and Informatics Vol. 3, No. 2 (2008), pp. 29-48

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Abstract. In this manuscript, we show that the Theorem 3.28 [2] is not correct and modify it.

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1. Introduction

The concept of BCK-algebra that is a generalization of set difference and propositional calculi was established by Imai and Iséki [3] in 1966. In [4], Jun et al. applied the hyper structures BCK-algebra. In 1934, Marty [5] introduced for the first time the hyper structure theory in the 8th congress of Scandinavian Mathematicians proceedings. In [1], Borzooei et al. introduced the generalization of BCK-algebra and hyper BCK-algebra, called hyper K-algebra. They studied properties of hyper K-algebra. In [2], Roudbari et al. investigated some properties of simple hyper K-algebras and classificaied simple hyper K-algebra of order 4. We show that the Theorem 3.28 [2] is not correct and modify it.

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2. Preliminaries

In this section, we mention theorems of [2] that are needed in the sequel.

Definition 2.1. [1] Let H be a non-empty set containing a constant " 0 " and " $\circ$ " be a hyperoperation on H. Then H is called a hyper K-algebra if it satisfies in the following properties:

(HK1): $(x \circ z) \circ (y \circ z) < x \circ y$,
(HK2): $(x \circ y) \circ z = (x \circ z) \circ y$,
(HK3): $x < x$,
(HK4): $x < y, y < x \Longrightarrow x = y$,
(HK5): $0 < x$.

for all $x, y, z \in H$, where $x < y$means $0 \in x \circ y$. Moreover for any $A, B \subseteq H$, $A < B$ if $\exists\, a \in A$, $\exists\, b \in B$ such that $a < b$.

Theorem 2.2. [2] *The following statements in a simple hyper K-algebra* $(H, \circ, 0)$ *hold.*

(1) $a \circ 0 = \{a\}, \forall a \in H \setminus \{0\}$,
(2) $a \in a \circ b$, *for all distinct elements* $a, b \in H$.

Theorem 2.3. [2] *Let* $(H, \circ, 0)$ *be a simple hyper K-algebra,* $|H| = n, a \in H \setminus \{0\}$ *and* $I = \{0, a\}$ *be a PIHKI of type 27(13, 18, 20). Then*

(1) *If* $0 \in x \circ y$ *then* $x \circ y = \{0\}$ *or* $\{0, a\}, \forall x \in H$,
(2) $a \circ b = \{a\}$ *for* $b \neq a$ *and* $b \in H$.

The Theorems 3.22, 3.23, 3.24, 3.25 and 3.26 [2] can be revise as follows:

Theorem 2.4. *Let* $(H_1, \circ_1, 0_1), (H_2, \circ_2, 0_2)$ *be two hyper K-algebras,* $H_1 = \{0_1, 1_1, 2_1, 3_1\}, H_2 = \{0_2, 1_2, 2_2, 3_2\}$ *and* $f : H_1 \to H_2$ *is a bijective map. If for all* $x, y \in H$, $\mid x \circ_1 y \mid \neq \mid f(x) \circ_2 f(y) \mid$ *then* f *is not an isomorphism hyper K-algebra*

Proof. It is clear. □

3. Main Results

In this section at first we show that Theorem 3.28 [2] as follows is not true, then we give a correct version of it. The authors in [2] claimed:
Theorem 3.28 [2]**:** There are 50 non-isomorphism hyper K-algebras of order 4 with simple condition such that they have exactly one positive implicative hyper K-ideal of order 2 and type 27(13, 18, 20).

The following example shows that in the list of hyper K-algebras $(H, \circ_i, 0)$, in [2], if $2 \circ_i 1 = \{1, 2\}$ or $3 \circ_i 1 = \{1, 3\}$, then $(H, \circ_i, 0)$ where (i=4, 7, 10, 12, 13, 15, 16, 17, 18, 19, 20, 21, 23, 24, 26, 28, 30, 32, 33, 35, 37, 38, 39, 40, 41, 43, 44, 46, 47, 49, 50) do not have any positive implicative hyper K-ideal of order 2 and type 27(13, 18, 20).

Example 3.1. The $(H, \circ, 0)$ with cayley table as follows be a hyper K-algebra,

H	0	1	2	3
0	{0}	{0}	{0}	{ 0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{1,3}	{3}	{0,1}

and $I = \{0, 1\}$ is not a positive implicative hyper K-ideal of type 27, because $(3 \circ 1) \circ 1 = \{0, 1, 3\} < I, 1 \circ 1 = \{0, 1\} < I$ but $3 \circ 1 = \{1, 3\} \nsubseteq I$. Similarly, $I = \{0, 2\}$ and $I = \{0, 3\}$ are not a positive implicative hyper K-ideal of H.

Theorem 3.2. *Let $H = \{0, a, b, c\}$ be a simple hyper K-algebra of order 4 and $I = \{0, a\}$ a positive implicative hyper K-ideal of type 27(13, 18, 20). Then $b \circ x = \{b\}$ such that $b \notin \{0, a\}$ and $x \neq b$.*

Proof. By Theorem 2.2(2), $b \in b \circ x$. Let $b \circ x \neq \{b\}$. Then, we have $b \circ x = \{a, b\}, \{a, b, c\}$ or $\{b, c\}$. If $b \circ x = \{a, b\}$ or $\{a, b, c\}$, then $a \in a \circ x \subseteq (b \circ x) \circ x < I$ and $x \circ x < I$ but $b \circ x \nsubseteq I$, which is a contradiction to I is a positive implicative hyper K-ideal of type 27. Also if $b \circ x = \{b, c\}$, then $(b \circ x) \circ b \neq (b \circ b) \circ x$, so HK2 is not satisfied. Because $c \in c \circ b \subseteq (b \circ x) \circ b$ but $b \circ b \subseteq \{0, a\}$ and $c \notin (b \circ b) \circ x \subseteq 0 \circ x \cup a \circ x \subseteq \{0, a\}$. Hence $b \circ x = \{b\}$. □

Theorem 3.3. *Let H be a hyper K-algebra of order 4, such that has three positive implicative hyper K-ideals of order 2 and type 27(13, 18, 20). Then H is a BCK-algebra.*

Proof. Let $H = \{0, 1, 2, 3\}$ hyper K-algebra such that $I_1 = \{0, 1\}, I_2 = \{0, 2\}$ and $I_3 = \{0, 3\}$ are positive implicative hyper K-ideals of order 2 and type 27. By Theorem 2.3(2), we have $a \circ b = \{a\}, \forall\, 0 \neq a \in I_i, a \neq b; i \in \{1, 2, 3\}$.
On the other, if I be a positive implicative hyper K-ideals of order 2 and type 27, then $x \circ x \subseteq I$. Since $(x \circ 0) \circ x < I$ and $0 \circ x < I$. So, $x \circ x \subseteq I_1 \cap I_2 \cap I_3 = \{0\}$, hence $x \circ x = \{0\}$ and H is a BCK-algebra. □

Now we modify Theorem 3.28 [2] as follows:

Theorem 3.4. *There are 22 non-isomorphism hyper K-algebras of order 4 with simple condition such that they have exactly one positive implicative hyper K-ideal of order 2 and type 27(13, 18, 20).*

Proof. Let $H = \{0, 1, 2, 3\}$ and suppose that the following table shows a probable hyper K-algebra structure of H.

$\circ$	0	1	2	3
0	a_{11}	a_{12}	a_{13}	a_{14}
1	a_{21}	a_{22}	a_{23}	a_{24}
2	a_{31}	a_{32}	a_{33}	a_{34}
3	a_{41}	a_{42}	a_{43}	a_{44}

Let $I = \{0, 1\}$ be a PIHKI of type 27. Then by Theorem 2.2(1), we have $1 \circ 0 = \{1\}, 2 \circ 0 = \{2\}, 3 \circ 0 = \{3\}$ and by Theorems 2.3(2), 3.2, $1 \circ 2 = 1 \circ 3 = \{1\}, 2 \circ 1 = 2 \circ 3 = \{2\}$ and $3 \circ 1 = 3 \circ 2 = \{3\}$. By HK3 and HK5 we have $0 \in a_{11} \cap a_{12} \cap a_{13} \cap a_{14} \cap a_{22} \cap a_{33} \cap a_{44}$. Also by Theorem 2.3(1), we get the only cases for $a_{11}, a_{12}, a_{13}, a_{14}, a_{22}, a_{33}$ and a_{44} are $\{0\}$ or $\{0, 1\}$. Therefore there is 2^7 hyperoperation on H. By HK2 and Theorems 2.4, 3.3, we get that there are 22 non-isomorphism hyper K-algebras of order 4 with simple condition such that they have exactly one PIHKI of type 27. Morever, if $I = \{0, 2\}$ or $\{0, 3\}$ be a PIHKI of type 27 of $H_i; 1 \leq i \leq 22$, then $(x \circ 0) \circ x < I$ and $0 \circ x < I$ but $x \circ x \nsubseteq I$. Thus $\{0, 1\}$ is the only PIHKI of type 27 of H_i.

H_1	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0}

H_2	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0,1}

H_3	0	1	2	3
0	{0}	{0}	{0,1}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0}	{2}
3	{3}	{3}	{3}	{0,1}

H_4	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0}

H_5	0	1	2	3
0	{0}	{0}	{0}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0}

H_6	0	1	2	3
0	{0}	{0,1}	{0}	{0}
1	{1}	{0}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_7	0	1	2	3
0	{0}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_8	0	1	2	3
0	{0}	{0}	{0}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_9	0	1	2	3
0	{0}	{0,1}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{10}	0	1	2	3
0	{0}	{0,1}	{0}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{11}	0	1	2	3
0	{0}	{0,1}	{0,1}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{12}	0	1	2	3
0	{0}	{0,1}	{0,1}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{13}	0	1	2	3
0	{0}	{0}	{0,1}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{14}	0	1	2	3
0	{0}	{0}	{0,1}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{15}	0	1	2	3
0	{0,1}	{0}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{16}	0	1	2	3
0	{0,1}	{0}	{0}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{17}	0	1	2	3
0	{0,1}	{0}	{0,1}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{18}	0	1	2	3
0	{0,1}	{0,1}	{0}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{19}	0	1	2	3
0	{0,1}	{0,1}	{0}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{20}	0	1	2	3
0	{0,1}	{0,1}	{0,1}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{21}	0	1	2	3
0	{0,1}	{0,1}	{0,1}	{0,1}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

H_{22}	0	1	2	3
0	{0,1}	{0}	{0,1}	{0}
1	{1}	{0,1}	{1}	{1}
2	{2}	{2}	{0,1}	{2}
3	{3}	{3}	{3}	{0,1}

$\square$

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