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Symmetry of Fullerene C_{60}

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Abstract. On analyzing the Euclidean graph of fullerene C_{60}, we have obtained the automorphism group of the Euclidean graphs of this molecule. It is proven that this group has order 120 and is isomorphic to $Z_2 \times A_5$, where Z_2 is a cyclic group of order 2 and A_5 is the alternating group on five symbols.

1 Introduction

Let $G = (V, E)$ be a simple graph. G is called a weighted graph if each edge e is assigned a non-negative number $w(e)$, called the weight of e. An unweighted graph G can be regarded as a weighted graph in which for all edges $e, f \in E(G), w(e) = w(f) = 1$. Euclidean graph of a molecule is a complete weighted graph, in which edges weighted by Euclidean distances.

An automorphism of a weighted graph G is a permutation g of the vertex set of G with the property that, (i) for any vertices u and v, $g(u)$ and $g(v)$ are adjacent if and only if u is adjacent to v; (ii) for every vertex v and edge e, $w(g(e)) = w(e)$. The set of all automorphisms of a weighted graph G, with the operation of composition of permutations, is a permutation group on $V(G)$, denoted $\mathrm{Aut}(G)$.

By symmetry we mean the automorphism group symmetry of a graph. The symmetry of a graph, also called a topological symmetry, accounts only for the bond relations between atoms, and does not fully determine molecular geometry. The symmetry of a graph does not need to be the same as (i.e. isomorphic to) the molecular point group symmetry. However, it does represent the maximal symmetry which the geometrical realization of a given topological structure may posses.

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In Refs. [1,2], it showed by Randic that a graph can be depicted in different ways such that its point group symmetry or three dimensional perception may differ, but the underlying connectivity symmetry is still the same as characterized by the automorphism group of the graph. However, the molecular symmetry depends on the coordinates of the various nuclei which relate directly to their three dimensional geometry. Although the symmetry as perceived in graph theory by the automorphism group of the graph and the molecular group are quite different, it showed by Subramanian[3] that the two symmetries are connected.

The automorphisms have other advantages such as in generating nuclear spin species, NMR spectra, nuclear spin statistics in molecular spectroscopy, chirality and chemical isomerism. The reader is encouraged to consult the papers by Balasubramanian[3–11] and Refs. [12-14] for background material as well as basic computational techniques.

Longuet-Higgins[15] showed that a more desirable representation of molecular symmetry is to use the nuclear permutation and inversion operations resulting in a group called Permutation-Inversion (PI) group. Balasubramanian[3] showed that the automorphism group of Euclidean graph of a molecule is the Permutation-Inversion group of the molecule.

In this paper we consider only weighted graphs. Throughout this paper, all groups and graphs considered are assumed to be finite. Our notation is standard and taken mainly from Refs. [16-21].

2 Results and discussion

In this section we first describe some notation, which will be kept throughout. Let G be a group and N be a subgroup of G. N is called a normal subgroup of G, if for any $g \in G$ and $x \in N, g^{-1}xg \in N$. If H is another normal subgroup of G such that $H \cap N = e$ and $G = HN = \{xy | x \in H, y \in N\}$, then we say that G is a direct product of H by N denoted by $H \times N$. A group with no proper non-trivial normal subgroup is called simple. Suppose X is a set. The set of all permutations on X, denoted by S_X, is a group which is called the symmetric group on X. In the case that, $X = \{1, 2, , n\}$, we denote S_X by S_n or $Sym(n)$.

Computations of the symmetry properties of molecules carried out with the use of GAP[22]. GAP is a free and extendable software package for computation in discrete abstract algebra. The term extendable means that you can write your own programs in the GAP language, and use them in just the same way as the programs which form part of the system (the "library"). More information on the motivation and development of GAP to date can be found on GAP web page that you find on http://www.gap-system.org. GAP

contains a large library of functions, which are important for the calculations of this paper. In this paper, we use freely these functions and the reader is encouraged to consult the manual of GAP[22] and Refs. [23, 24].

A permutation matrix is a matrix that has exactly one 1 in each row or column and 0s elsewhere. Permutation matrices are the matrix representation of permutations. In general, for a permutation σ on n objects, the corresponding permutation matrix is an n-by-n matrix P_σ is given by $P_\sigma = [xij], xij = 1$ if $i = \sigma(j)$ and 0 otherwise. We can see that $P_\sigma P_\pi = P_{\sigma\pi}$, for any two permutations σ and π on n objects, $P_{(1)(2)\cdots(n)}$ is the identity matrix, and, permutation matrices are orthogonal matrices, a square matrix whose transpose is its inverse. Thus $(P_\sigma)^{-1} = P(\sigma^{-1})$. If P denotes the set of all permutation matrices then P is a group under product of matrices. This group has order n! and is isomorphic to Sn, the symmetric group on n symbols.

The adjacency matrix of a weighted graph is defined as: $A_{ij} = w_{ij}$, if $i \neq j$ and vertices i and j are connected by and edge with weight w_{ij}; $A_{ij} = v_i$, if $i = j$ and the weight of the vertex i is v_i, and, $A_{ij} = 0$, otherwise. Note that A_{ii} can be taken as zero if all the nuclei are equivalent. Otherwise, one may introduce different weights for nuclei in different equivalence classes and the same weight for the nuclei in the same equivalence classes. A permutation of the vertices of a graph belongs to its automorphism group if it satisfies $P^tAP = A$, where Pt is the transpose of permutation matrix P and A is the adjacency matrix of the graph under consideration. There are n! possible permutation matrices for a graph with n vertices. However, all of them may not satisfy the above relation. It is important to note that the automorphism group of a graph depends only on the connectivity of the graph and does not depend on how the graph is represented in three dimensions. That is, a graph, in general, can be represented in different ways in three dimensions such that two representations could yield different three-dimensional symmetries and yet their automorphism groups are the same since the latter depends only on which vertices are connected in the graph.

Consider C_{60} fullerene, Figure 1, to illustrate the Euclidean graph and its automorphism group. It should be mentioned that, we apply Gussian 98 and HyperChem in Tables I to compute the Euclidean distances. Also, we don't have to work with exact Euclidean distances in that a mapping of weights into a set of integers suffices as long as different weights are identified with different integers. In fact the automorphism group of the integer- weighted graph is identical to the automorphism group of the original Euclidean graph. To illustrate let us use a Euclidean edge weighting for fullerene C_{60}, as in Tables I. Suppose A is the 60×60 matrix defined by Tables I. Using a GAP program[24], we can compute all the permutation matrices P and Q such that $P^tAP = A$.

Not all 60! permutations of the vertices C_{60} belong to the automorphism group of its weighted graph since the weights of all the edges are not the same. For example, the permutation (1,2,3,4,5,6,7) does not belong to the automorphism group since the resulting graph does not preserve connectivity. Let X denotes the set of all solutions of matrix equations $P^tAP = A$. Set $X_1 = \{\alpha \in S_{60} | P_\alpha \in X\}$. Then X1 is automorphism group of the Euclidean graph of C_{60}. We now write a GAP program to find the structure of this group, as follows: A GAP Program for Computing the Structure of the Automorphism group of the Euclidean Graphs of C_{60} and C_{80}

$G := Group(X);$
$R := NormalSubgroups(G);$
$I := Intersection(R[2], R[3]);$
$Size(G); Generators(R[2]);$
$Generators(R[3]); IsSimple(R[2]);$
$IsSimple(R[3]);$

After running this program, we can see that G has exactly two proper non-trivial normal subgroups $N_1 = R[2]$ and $N_2 = R[3]$ which intersects trivially. Therefore, G is isomorphic to the direct product $Z_2 \times A_5$, where Z_2 is a cyclic group of order 2 and A_5 is the unique simple group of order 60. We now consider the following permutations:

x_1:=(1, 7, 2, 4, 8)(3, 17, 5, 10, 12)(6, 21, 13, 16, 27)(9, 26, 11, 18, 28)
(14, 36, 25, 23, 34)(15, 37, 19, 22, 33)(9, 26, 11, 18, 28)(14, 36, 25, 23, 34)
(15, 37, 19, 22, 33)(20, 48, 38, 30, 43)(24, 44, 29, 35, 42)(31, 54, 40, 47, 50)
(32, 55, 41, 46, 51)(39, 56, 49, 45, 58)(52, 60, 53, 59, 57),
x_2:=(1, 11)(2, 5)(3, 6)(4, 19)(7, 25)(8, 13)(9, 14)(10, 29)(12, 28)(16, 20)(17, 38)
(18, 40)(21, 41)(22, 39)(23, 32)(26, 42)(27, 33)(30, 31)(35, 53)(36, 49)(37, 50)
(44, 51)(45, 46)(47, 52)(48, 57)(54, 58)(55, 60)(56, 59),
y_1:=(1, 60)(2, 59)(3, 58)(4, 57)(5, 56)(6, 54)(7, 53)(8, 52)(9, 51)(10, 49)(11, 55)
(12, 45)(13, 47)(14, 44)(15, 43)(16, 50)(17, 39)(18, 41)(19, 48)(20, 37)(21, 40)
(22, 38)(23, 42)(24, 34)(25, 35)(26, 32)(27, 31)(28, 46)(29, 36)(30, 33),
z_1:=(1, 60)(2, 57)(3, 58)(4, 59)(5, 49)(6, 51)(7, 52)(8, 53)(9, 54)(10, 56)(11, 50)
(12, 39)(13, 41)(14, 43)(15, 44)(16, 55)(17, 45)(18, 47)(19, 42)(20, 34)(21, 46)
(22, 35)(23, 48)(24, 37)(25, 38)(26, 31)(27, 32)(28, 40)(29, 33)(30, 36),
z_2:=(1, 10, 9)(2, 18, 15)(3, 4, 16)(5, 21, 24)(6, 7, 23)(8, 22, 14)(11, 17, 30)(12, 35, 20)
(13, 36, 31)(19, 26, 46)(25, 37, 45)(27, 47, 29)(28, 48, 32)(33, 55, 39)(34, 56, 40)
(38, 44, 52)(41, 43, 59)(42, 54, 53)(49, 51, 60)(50, 58, 57)

We now apply our GAP program. By this program, $\{x_1, x_2\}$ is a generating set for G. Suppose $N_1 =< y_1 >$ and $N_2 =< z_1, z_2 >$. Since N2 is a simple group of order 60 and A_5 is the unique simple group of this order, $N_2 \times A_5$.

Therefore, the automorphism group of the Euclidean graph of fullerene C_{60} has order 120.

3 Conclusions

We can use this method for computing with small groups. When the order of the group is large, the commands "Groups(X)" and "NormalSubgroups(G)" are interrupted. In this situation, one can find a characteristic subgroup N of G and then compute all the normal subgroups of N and G/N. This method is usually very useful for calculating the normal subgroup of the groups of small order[22,23].

On the other hand, our calculations with GAP and calculations done by Balasubramanian[3–11], Hao-Xu[12], Ivanov[13] and Ivanov-Schrmann[14], show that the automorphism group of the Euclidean graph of every molecule is trivial or have an even number of elements.

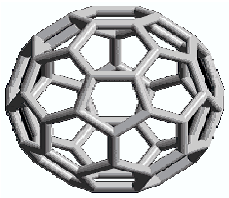

Fig. 1. The Fullerene C_{60}

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Table 1. The Euclidean Distances for C_{60}

0	1.4	1.4	1.4	2.4	2.4	2.2	2.2	2.4	2.4	2.8	3.5	3.6
3.5	3.5	2.8	3.5	3.6	4	4.5	1.4	0	2.4	2.2	1.4	2.8
2.2	1.4	3.6	3.5	2.4	2.4	2.4	4	4.4	4	3.5	4.4	3.5
4.7	1.4	2.4	0	2.4	2.8	1.4	3.5	3.5	1.4	2.8	2.4	4.5
4	2.2	2.2	2.4	4.5	4	3.6	3.5	1.4	2.2	2.4	0	3.5
3.6	1.4	2.2	2.8	1.4	4	3.5	4.4	4.4	4	2.4	2.4	2.4
5	5.3	2.4	1.4	2.8	3.5	0	2.4	3.5	2.4	4	4.5	1.4
2.8	1.4	3.6	4.4	4.7	4.5	5.3	2.2	4	2.4	2.8	1.4	3.6
2.4	0	4.4	4	2.2	4	1.4	4.7	3.5	1.4	2.2	3.5	5.3
5	2.4	2.4	2.2	2.2	3.5	1.4	3.5	4.4	0	1.4	4	2.4
4.4	2.4	4	5.2	5	3.6	1.4	2.8	5.2	6	2.2	1.4	3.5
2.2	2.4	4	1.4	0	4.4	3.5	3.6	1.4	2.8	5	5.2	4.4
2.4	4	4.4	5.6	2.4	3.6	1.4	2.8	4	2.2	4	4.4	0
2.4	3.5	5.3	5	2.2	1.4	1.4	4.7	3.5	4.4	3.5	2.4	3.5
2.8	1.4	4.5	4	2.4	3.5	2.4	0	4.7	4.5	5.3	4.4	3.6
1.4	2.8	1.4	5.6	5.3	2.8	2.4	2.4	4	1.4	1.4	4.4	3.6
3.5	4.7	0	4	2.2	2.4	3.5	4.5	5.3	5.6	1.4	2.8	3.5
2.4	4.5	3.5	2.8	4.7	2.4	1.4	5.3	4.5	4	0	2.4	5.6
6	5.3	2.8	4.7	4.4	5.9	3.6	2.4	4	4.4	1.4	3.5	4
2.8	5	5.3	2.2	2.4	0	4.4	5.2	5.6	4.7	5.9	2.2	4.4
3.5	4	2.2	4.4	3.6	1.4	5.2	5	2.2	4.4	2.4	5.6	4.4
0	1.4	3.5	6	5.2	2.8	1.4	3.5	4.4	2.2	4	4.4	2.2
5	5.2	1.4	3.6	3.5	6	5.2	1.4	0	2.4	5.6	4.4	4
2.4	2.8	4	2.4	2.4	4.7	3.5	3.6	4.4	1.4	1.4	4.5	5.3
5.6	3.5	2.4	0	4	2.2	5.3	4.5	3.5	3.5	4.5	2.4	4.5
5.3	1.4	2.4	4.7	2.8	5.3	2.8	4.7	6	5.6	4	0	2.4
6	6.5	3.6	4.4	4	2.4	5.3	5	2.8	4	3.5	1.4	5.6
4.7	5.9	5.2	4.4	2.2	2.4	0	6.3	6	4	3.5	3.6	5
2.2	2.4	5.2	4.4	4.4	5.6	1.4	4.4	2.2	2.8	4	5.3	6
6.3	0	2.4	4.5	4.7	3.5	5.3	4	2.4	6	5.6	3.5	5.3
2.8	5.9	4.4	1.4	2.4	4.5	6.5	6	2.4	0	4	4.4	4.7
2.8	5.3	5.6	2.4	3.6	4.5	2.4	5.9	4	5.6	6	5.3	3.5
1.4	1.4	6.5	6.5	4.4	5.2	4.4	3.5	6	5.2	4	5	3.5
2.2	6	5.6	6.5	5	4	2.2	3.6	1.4	6.5	5.6	4	5
3.6	3.5	5.6	4.4	4.4	5.2	2.4	2.2	5.3	6	6.3	4	2.8
1.4	4.4	2.2	5.9	4.7	4.5	5.3	3.5	4.7	5.3	3.5	5.6	6
2.4	4	4.5	6.5	6	2.4	1.4	2.8	5.9	4.4	4.7	2.8	4.4
3.5	4.4	5.2	2.2	3.5	5	4	5.2	6	2.2	3.6	1.4	4
5	6	5.6	6.5	1.4	3.6	4.4	4	5.3	3.6	4.7	5.9	2.4
2.8	5.6	4	5.6	2.4	4.5	6.5	6.3	5	1.4	3.5	6	6.8
4.4	3.6	5.3	4	4	5.6	2.8	2.4	5.9	4.7	5	1.4	3.5
6.3	6.5	5.6	2.4	4.5	5.2	6.5	4	2.8	4.7	4.4	2.4	4.5
3.6	2.4	5.6	5.3	3.5	1.4	1.4	5.3	6	5.9	4	5.6	3.5
5.3	4.7	4.5	4	5.6	3.5	2.8	6	5.3	4.4	5.9	2.4	5.3

Table 1. (Continued)

3.5	2.4	3.6	5.3	6.5	6.5	1.4	1.4	4.7	5.6	4	4.5	5.9
4.4	5.3	6	2.8	3.5	5.3	6.5	6.5	3.6	2.4	2.4	5.3	3.5
4	4.4	4	4.5	4.4	4.4	4.4	4	4.7	4.7	5.3	5.3	5
5.2	5.3	5	5.2	5.3	6	5.6	4.4	5.2	5	5.3	3.5	4
3.6	2.8	4.5	5.6	5.9	5.6	4	4.4	6	5.2	5	4.5	6
5.3	4.7	4.4	3.6	3.5	4.4	5.3	5.3	4.7	4	4	4.4	4.4
5.6	6	5.3	5.6	6	5.3	5.2	5	2.8	3.5	3.5	4.7	5.2
3.6	4	4.4	5.6	4.5	5.6	5.9	5.2	5	4.5	4	4.4	6
6.5	6.3	5.3	6	5.6	5.3	2.2	4.7	4	2.4	3.5	5.9	5.6
5	3.6	4.4	6.5	6	5.6	3.5	5.2	4.4	5.6	5.2	4.4	3.5
3.5	5.9	5.6	4.5	2.8	4.4	4	3.6	5.3	6	6	6.3	6.5
4.5	4.4	4	2.4	4	4.4	5.6	5	2.4	2.8	3.6	6	5.3
6.3	6.5	4.4	4	4.7	3.5	3.5	5.6	6.8	6.5	3.6	5	5.2
6	4	2.8	2.4	2.4	5.3	6	6.5	6.3	3.5	3.5	5.6	4.4
4	4.7	6.5	5.9	4.5	3.5	2.4	2.4	5.2	5.6	5.9	5.6	4.4
2.8	3.6	4	6.3	6.5	4.5	5.3	6	6	5	5.2	2.4	2.2
2.2	4	6	4	4.7	5.3	5.9	3.5	5	5.6	6	5.6	3.5
3.6	4.4	6.5	6.3	6.5	5.9	6	5.3	4.5	2.2	5.6	5	3.5
2.4	5.3	4.7	4	4.4	5.2	6.5	6.5	6.3	3.5	4.4	3.6	4
5.6	6	6.5	3.6	2.4	1.4	1.4	5.3	6.5	6.8	6.5	2.2	2.2
5.9	4.4	3.6	4	6.3	5.6	5.6	6.5	6.3	6	1.4	4.5	3.5
1.4	3.5	6.5	6	5.2	2.4	3.5	6.8	6	5.3	2.4	5	4
6	5	4	2.4	4	6.5	6.3	5.3	2.4	3.6	2.8	2.4	5.9
6.5	5.6	6.5	6.8	4.7	3.5	3.5	5.3	4	2.8	1.4	5	6.3
6.5	6	3.6	2.4	2.4	2.8	6.5	6.8	4.7	5.9	6.5	5.6	4
4.4	3.5	2.2	1.4	2.8	6	5	5.6	5.9	5.3	2.4	4	4.7
6.5	6.3	3.5	4.4	5.2	6.5	5.6	6	1.4	3.6	4.4	5.9	5.6
1.4	2.4	4	6.5	5.3	6.5	6.8	4.4	3.6	4	2.2	2.2	5.9
6.9	6.8	1.4	1.4	2.2	4.4	6.5	3.5	4.5	5.6	6.5	3.5	5.2
6	6	5.3	2.4	2.4	3.5	6.8	6.5	6.8	6.5	6.5	5.9	4.7
1.4	6	5.2	3.5	1.4	5.6	4.5	3.5	4	5	6.8	6.8	6.5
2.4	3.5	2.4	6.5	5.6	4.7	2.8	3.6	6.8	6.5	5.3	1.4	4
2.4	1.4	5.6	6.3	5.9	6.8	6.9	4	2.2	2.2	0	2.4	3.5
5.3	6.3	2.2	3.5	5	6.8	4.5	6	6.5	5.2	4.4	2.8	1.4
2.2	6.5	6.8	6.9	2.4	0	1.4	3.6	6.8	4.4	5.3	6.3	6.3
2.4	4.4	5.3	6.5	5.9	1.4	2.8	4	6.9	5.9	6.5	3.5	1.4
0	2.4	6.5	5.2	6	6.5	5.6	1.4	3.5	4.5	6.8	6.5	2.4
4	5	6.8	5.3	6	5.3	3.6	2.4	0	5.6	6.5	6.8	6.5
4	1.4	1.4	2.4	6.8	6.9	4	5.6	6.3	5.9	3.6	4.4	6.3
6.8	6.5	5.6	0	5.3	4.4	2.4	2.4	6.3	5.3	4.4	2.8	4
6.9	6.5	5.9	1.4	4	2.8	2.2	4.4	5.2	6.5	5.3	0	1.4
3.5	6.5	6	6.8	6.9	3.5	2.4	4.4	2.2	1.4	5.3	6.8	6.5
3.5	5.3	6	6.8	4.4	1.4	0	2.2	6	6.5	6.9	6.8	2.2
1.4	5.3	3.5	2.4	4.4	6.5	6	5	6.3	6.5	6.5	2.4	3.5

Table 1. (Continued)

2.2	0	4.5	6.8	6.5	6	1.4	2.2	6.5	5.2	4.4	2.8	5.6
4.7	6.8	6.3	5.6	4	2.4	6.5	6	4.5	0	5	3.5	2.2
4.7	5.6	6.5	6.9	6.8	2.8	2.2	1.4	4.5	2.4	1.4	1.4	6.3
6	6.5	6.8	5	0	2.2	3.5	6.9	6.8	2.8	4.7	5.6	6.5
5.9	5.6	6	6	6	5.6	5.9	5.6	6.5	6.3	6.5	6.5	6.5
6.5	6.3	6.5	6.8	6.8	6.8	6.9	5.3	4.7	5.3	5.6	6.5	6.3
6.5	6	6	5.6	5.9	6.8	6.5	6.3	6.5	6.8	6.5	6.5	6.9
6.8	5.6	5.9	6.5	6.5	5.2	5	5.6	5.9	6.3	6.5	6.8	6
6	6.8	6.5	6.3	6.5	6.9	6.5	6.8	6.5	6	5.6	5.3	6
5.3	5.3	4.7	6.8	6.5	6.3	6.5	6.8	5.9	5.6	6	6.9	6.5
6.5	6.8	4.4	4	5.3	5.9	6.3	6.5	6.8	6.5	5.2	5	5.6
6.5	6	6.5	6.8	6.9	6	6.3	6.8	6.5	4.7	5.3	6.5	6.8
5	5.2	6	6.5	5.6	6	6.5	5.6	5.3	6.9	6.8	6.5	5.9
6.8	6.3	6.5	6.3	5.3	4.7	4.5	6.5	5.9	5.6	4.5	6.5	5.9
5.6	6.8	6.9	5.3	5.3	6	6.8	6	6.5	6.5	5.6	4.5	4.5
4.7	6.8	6.5	6.3	5.3	6	5.3	5.3	6.9	6.8	5.6	5.9	6.5
6.5	6	6.8	6.5	6	6.5	6.8	6.5	4.4	4	4.7	5.3	6.5
6.8	6.9	5.3	5.6	6.5	6	5.6	6.3	6.8	5.9	6.5	6.8	6.5
5.9	5.3	5.2	4.4	4.4	4	6.9	6.8	6.5	6	6.5	5.6	5
5.2	6.8	6.3	6	6.5	4	4.4	6	6.5	5.6	6	6.5	6.8
5	5.2	6	5.9	5.3	6.8	6.9	6.8	5.6	6.5	6.5	6.3	5
3.5	3.5	4	6.9	6.8	6.5	5.3	5.2	4.4	4.4	6.8	6.5	5
5.6	6.3	6	5.2	6.5	6	3.6	2.8	4.5	5.3	6.5	6.8	6.9
6.5	4.4	4	4.7	6.3	5.6	6	6.5	6.8	5.3	5.6	6.5	5.9
4.5	5.6	6.8	6.9	4	4.4	5.3	6.3	5.3	6	6.5	4.7	4.5
6.8	6.5	5.9	5.3	6.5	5.6	6	5.3	6.3	6.9	6.8	3.5	3.5
4.5	5.6	5.9	6.5	6.8	4.5	4.7	6.5	6	5.3	5.6	6.5	5.3
6	6.5	6.8	6.5	6	4.4	3.6	4	4.4	6.8	6.9	6.8	5.3
5.9	6	5.2	5	6.5	6.5	5.6	6.3	6.5	5.3	4	3.5	6.3
5.6	5	3.5	6.3	5.6	5	6.5	6.8	4.4	4.4	5.2	6.5	5.2
6	6	6.9	6.5	5.3	4.5	5	4	3.6	2.8	6.8	6.5	6
5.6	6.3	4.7	4	4.4	6.5	5.6	5.3	5.9	2.8	3.6	5.6	6.3
5.3	6	6.5	6.9	4	4.4	5.3	5.3	4.5	6.5	6.8	6.5	4.7
5.9	6	5.6	3.5	5	6.5	6.8	3.6	4.4	5.3	6.5	4.4	5.2
6	4	3.5	6.5	6.3	5.6	4.4	6	5	5.2	6.8	6	4.4
3.5	5.6	4.7	4	2.4	6.5	6	5.2	5.9	6.5	4	3.6	4.4
6.3	5	5.3	5.6	6.8	6.8	5.6	4.7	4	2.8	2.4	2.4	6.5
6.5	6	4.7	5.6	4.5	3.5	3.5	6	5.3	4.5	5.3	6.5	6.9
6.3	5.6	3.5	2.4	2.8	3.6	6.5	6.8	6.5	4.5	5.3	5.3	4.4
4	6	5.9	4.7	5.6	5.3	6.5	6.8	6.5	2.2	2.2	3.5	5
5.6	6.3	6.5	3.5	4	6	5.2	4.4	5	6	4.4	5.2	2.4
2.4	4.7	5.6	5.9	6.5	6.8	6.8	3.5	3.5	4.5	5.6	4.7	6
6.5	6.5	4.5	5.3	6	5.3	6	4.5	2.8	2.4	6.5	6	5.2
3.5	5.6	4.7	4	6.3	6.5	3.6	4	5	5.9	4.4	5.6	5.3

Table 1. (Continued)

5.2	3.5	2.4	2.8	6.8	6.5	6	4.5	5	4	3.6	6.5	6.3
4	4.7	5.6	5.6	4.4	5.9	5.3	4	2.4	3.5	4.4	6.8	6.9
6.8	6	4.4	3.6	4	6.5	5.9	5.2	6	6.5	5.3	5	6.3
5.6	2.4	4	5.9	6.5	4.4	5.2	6	6.8	3.6	4.4	5.3	4.4
3.5	6.3	6.5	6	4	5.6	5.2	5	6	6.8	6.5	5.9	2.2
1.4	2.4	4	6	6.5	6.3	3.5	4.4	5.3	4.4	3.6	5.2	5.6
5.3	5.9	4.4	5.6	5.6	4	6.3	6.5	3.6	5	4.7	6.8	6
2.8	2.4	4	6.5	5.2	4.5	2.4	5.3	5.6	4.4	5.9	5	3.6
6.5	6.3	4	5.6	4	6.5	5.2	2.4	2.8	4.7	6.8	6	3.5
1.4	5	4	5.6	5.2	3.6	5.3	4.4	3.5	6.3	6	4.4	2.2
2.4	5.9	6.5	6.5	4.4	6	4	5.6	5.2	4.4	6	5	4.4
6	4	3.5	6.5	5.6	5.2	2.2	3.5	6.5	6.8	6.3	3.6	5.3
5	6.3	5.3	6	5.3	4.5	6.5	6	4.7	5.6	4.5	3.5	6.5
5.9	6.8	5.6	4.7	3.5	4	2.4	6.8	5.9	5	5.2	5.6	4
6	6.3	3.5	4.4	5.3	3.6	6.5	4.4	6	6.5	5.9	4.4	2.2
2.4	6.8	6.8	5.2	5	6	4.4	5.6	6.5	3.5	4	6	4.4
6.3	3.6	5.3	6.8	6.5	5.2	2.2	3.5	6.5	6.9	5.3	4.5	5.3
6	3.5	4.5	5.6	4.7	6	6.5	3.5	4	2.4	4.7	5.6	6.5
5.9	6.8	2.4	4	6	6	5.2	6.5	5.2	4.4	6.8	6.5	5
6.3	4.4	6.3	5	3.5	4	5.6	6.9	6.5	3.5	2.2	5.6	5.3
5	6.3	4.4	4	6.5	5.9	5.2	6.5	3.6	5.6	4	3.5	4.4
6	6.8	6.8	2.4	2.2	5.9	5.3	5.6	6.5	4.4	4.7	6.3	5.6
6	6.8	4	5	3.6	4.5	5.3	6.5	6.5	6.9	2.8	3.5	5.6
4.7	5.9	6	4	5.3	5.3	4.5	6.5	6.5	4.4	3.5	2.8	5.6
6.3	6.8	5.3	6.5	3.6	5	6	5.3	6.5	5.6	5.3	6.5	4.7
4.5	6.8	5.9	6	3.5	4.5	6.8	6.9	6.5	4	5.3	5.6	6.5
5.6	6.5	5.3	5.9	6.8	4.5	4.7	6.5	5.3	6.5	4	5.3	6.9
6	3.5	4.5	6.3	6.8	6	6.5	5.2	6	6.3	5	6.5	6.8
4.4	5.2	5.6	6.9	6.5	4	3.5	4.4	6.3	5	5.3	3.6	5.6
6.3	5	5.3	6.5	5.2	5.9	6.5	4	4.4	6	6.8	6.8	4.4
3.5	3.6	5.6	4	6	4.4	5.9	6.5	5.6	5.3	6.8	6	5.6
6.3	4.7	4.4	6.5	6.5	6.9	5.3	4.5	4	5	3.6	6.5	5.3
5.6	6	5.9	4.7	6.5	6.5	4.5	5.3	5.3	4	6.8	5.3	6.5
6.3	5.6	4.4	3.5	2.8	6.9	6.5	6.5	6	6.3	6.8	5.2	5.6
6.5	6	6.5	6.9	5	5.2	4.4	5.3	5.9	6.8	6.3	6.8	4
4.4	6.3	5.6	6.5	6.5	5	6	5.9	5.3	6.8	6.8	5.2	4.4
4	6	6.5	6.9	5.6	6.5	4.4	5.2	6.5	5.9	6.8	6.3	5.6
6.5	5.6	5.3	6.9	6.5	6	4.4	4.7	6.5	6.8	6.8	5	6
5.3	6	6.5	6.8	6	6.5	6.5	5.6	6.8	6.9	5.3	6	5.9
6.8	6.3	4.7	4.5	5.3	6.5	5.6	5.3	4	6.5	6.5	6	6.8
6	5.3	6.9	6.8	5.6	6.5	5.3	6.5	5.6	4.5	4.7	5.9	6.8
6.3	4.5	3.5	6.5	6.3	6.8	5.9	6.5	6.9	5.3	5.6	6.5	5.6
6.8	5	6	6.8	6.5	6	4.4	4.7	6.5	6.5	6.3	6.5	6.5
5.6	6.8	6.8	5.3	5.9	6	5	6.9	5.6	6.5	6.5	6	5.2

Table 1. (Continued)

4.4	4	6.8	6.3	6.5	6.8	6.3	6	6.9	6.5	6	6.5	5.6
5.2	6.8	6.3	6.8	5.9	5.3	5	5.2	4.4	6.5	5.6	6.8	6.5
6.5	6.9	6	5.9	6.8	6.5	6.3	6.8	5.6	6	5.3	5.3	5.6
6.5	6.5	6.5	4.7	4.4	6.8	6.5	6.9	6.5	6.3	6.8	6	6
6.8	6.3	6.5	5.2	5.6	6.5	6.5	6.5	5.2	5.6	5.9	6	6.8
6.9	6.5	6.5	6.8	6.3	6.5	6.8	5.9	6	6.5	6.5	6.5	5.6
5.3	5.6	6	5.3	6	5	6.9	6.8	6.8	6.8	6.5	6.5	6.5
6.5	6.5	6.5	6.3	6	5.9	6	6	6.3	6	5.9	5.6	5.2
6	4.4	3.5	1.4	5.3	6.8	6.9	6.5	3.5	2.2	0	1.4	6.5
6.8	4.4	6	6.5	5.3	2.4	3.5	6.5	5.3	4.5	2.4	4.4	6.9
6.8	6	2.2	3.5	1.4	0	6	6.5	5.3	6.5	6.8	4.4	1.4
2.2	5.2	6.5	6.8	6.8	2.8	3.5	2.2	1.4	4.7	6.9	6.5	6
0	1.4	6.3	5	4	2.4	5.3	4.5	4.4	5.9	6.5	6.9	4
2.4	1.4	2.2	5.6	6.8	6.8	6.5	1.4	0	5.6	4	2.8	3.6
5.9	5.3	2.8	1.4	2.4	4	6.9	4.4	5.3	6.5	6.5	2.8	4.4
5.3	6.3	5.6	0	2.4	3.6	6.8	5.6	6.3	1.4	2.8	4	5.6
6.5	2.2	3.5	5.2	6.9	4.7	6	6.5	5	4	2.4	0	1.4
6.3	6.5	6.8	2.2	4	5	6.3	5.9	1.4	2.4	4.4	6.8	5.6
6.5	6.8	4	2.8	3.6	1.4	0	5.6	6.5	6.5	6.5	6.9	6.8
5.9	1.4	5.3	4.4	2.8	2.8	6.5	5.3	4.4	2.4	3.6	6.8	6.3
5.6	0	3.6	2.4	6.8	5.9	5.3	3.6	4	6.8	6.5	5.6	2.2
4.4	2.4	1.4	5.3	5.9	5.6	6.5	6.5	3.6	0	1.4	6.9	6.5
6	4.4	2.8	6.5	6	4.7	1.4	5.2	3.5	2.2	4.5	5.3	6.3
6.8	6.5	2.4	1.4	0	6.8	6.8	6.5	5.3	2.4	6	5.2	4
2.4	6	4.5	3.5	3.5	4.4	6.5	6.5	6	1.4	2.4	1.4	6
6.8	6.9	6.5	2.4	4.5	3.5	2.4	4	6.8	6	5.2	1.4	2.4
6.5	5.6	4.7	1.4	4.4	3.5	4.4	5.6	6.3	6.8	4.7	2.8	2.4
3.5	5.9	6.5	6.5	6.3	2.4	1.4	5	3.6	2.4	4	5.6	5.3
3.5	4.7	5.6	6.5	5.6	2.4	2.8	4.4	6.5	5.9	6.3	6.5	3.6
2.4	4	2.4	1.4	5	6	6	5.6	4	3.5	2.2	5.9	6.5
6.8	6.8	4.4	2.2	1.4	2.4	6.5	6.5	3.6	5.3	5.9	5.6	2.8
4	4.7	2.8	2.4	2.2	6.5	6	6.5	6.9	5.2	1.4	2.2	3.5
6.8	6.5	2.4	4.5	5.3	6.3	4	5	4	2.4	2.8	3.5	6.8
5.2	6	6.8	6	2.4	3.5	4.5	6.5	6	1.4	3.5	4.4	6.5
4.7	5.6	2.4	2.4	3.6	5	6.8	3.5	4.5	6	6.8	4	5.2
6	5.6	4.7	1.4	1.4	2.4	6.5	6	6.5	6.5	6.5	6.5	5.6
3.5	5.6	5	4.4	3.6	6	4.7	4	3.5	4	6	5.9	5.3
2.2	2.8	2.4	6	6.5	6.8	6.3	3.5	4.7	4	3.6	4.4	6.5
5.6	5	2.4	2.8	6	5.3	4.5	2.2	4	3.5	5.2	6	6.5
6.5	4.5	4	3.6	4	5.3	6.3	5.9	5.6	2.8	2.4	5.2	4.4
3.5	3.5	4.7	4.5	5.9	4.7	4.5	3.5	5.6	6.3	6.5	6.5	4.4
3.5	2.4	2.8	6	6	4	5.3	5.6	5	2.4	3.6	6.5	5.6
5.3	4	4.7	6.5	6.3	5.9	3.5	4.4	2.8	2.4	5.3	5.6	5
6	6	4	1.4	2.4	4	4.5	5.3	6	6	3.6	4	5.2

Table 1. (Continued)

6.3	5.3	5.6	5.9	4.4	3.5	3.5	2.8	2.4	5.2	5.3	5.6	3.6
3.5	4.4	5.2	6.5	4	4.7	6	6.5	4.4	5	5.6	5.3	4.5
2.2	2.4	2.8	6	5.3	5.9	4.4	3.5	4	4.4	6.5	5	5.6
6.5	6	3.6	4	4.7	5.9	5.3	2.2	3.5	4	6	4.5	5.3
6.3	6	6	5	4.5	5.9	5.6	5.3	4	5.2	4	3.6	4.5
4.7	5.2	5.6	5.3	3.5	2.4	2.8	5	5.3	5.9	6	5.3	4.4
4.4	5	5.6	5.6	5.3	5.3	4	3.5	4.4	4	3.5	4.4	4.5
4.7	5.3	4.5	4.7	4.4	6	5.6	5.9	6.3	5.2	4	3.6	4
5.6	5.3	3.5	4.5	4.7	5.2	3.5	4.4	5.6	5.3	5.6	5.2	5.3
5.3	5.3	5.6	5	5	4.4	4.4	4.7	4.5	4.4	4.7	4.5	4.4
4.5	6	6.5	6.3	1.4	2.2	3.5	5.2	4.7	5.6	5.9	2.4	2.8
5.6	5	4	4	5.3	3.6	4.4	3.5	5.2	6.3	6.5	2.4	3.5
4.5	6	4	5	5.6	2.8	2.4	5.9	5.6	4.7	3.6	5.3	4
4.4	3.5	1.4	2.4	3.6	6.5	6.8	6.5	5.6	3.5	2.4	2.8	6
5.3	4.4	5.3	5.9	4.5	4	5.6	4.7	4.4	2.4	1.4	2.4	6.5
6.5	6	4.7	4	2.8	2.4	6	5.6	3.5	4.5	5.3	4.7	3.5
5.3	4.5	6.5	6.5	5	4	3.6	2.4	1.4	1.4	6	6	5.2
4	5	3.5	2.2	2.2	5.2	4.4	3.5	4.4	6.5	5.6	3.6	2.4
5.3	4.5	3.5	1.4	5.9	5.3	4.4	5.3	6	2.8	2.4	3.5	5.6
4	4.5	4.7	6	4.7	2.4	1.4	5.9	5.3	4.4	2.4	5.3	4.5
3.5	5.6	6	2.4	2.8	4	5.3	3.5	4.7	4.5	1.4	1.4	4
5	5.6	6.3	6.5	6.5	2.2	2.2	3.5	5	4	5.2	6	6
3.5	4.4	5.2	4.4	2.4	4.4	5.6	6	2.8	4	4.7	6	2.8
4	4.7	2.4	1.4	5.3	5.3	4.5	2.4	4.5	3.5	3.5	1.4	3.5
5.3	6	4	5	5.6	6.5	2.4	3.5	4.5	3.6	2.4	5.6	5.9
5.3	2.8	4.7	4.4	4	0	2.2	4.4	5.2	4.7	5.6	5.9	6.3
1.4	2.2	3.5	4	2.8	5	5.6	5.3	2.4	4	4.4	3.6	2.2
0	2.8	4	6	6.5	6.3	5.9	2.2	1.4	2.4	5.2	4.4	4.4
5.3	5.6	3.5	3.6	5	4	4.4	2.8	0	1.4	6	6	5.2
4	3.6	2.4	1.4	5.2	5	2.2	3.5	4.4	4	2.2	4.4	3.5
5.2	4	1.4	0	5.6	5.3	4.4	2.8	4.4	3.5	2.2	5	5.2
1.4	2.4	3.6	4.4	2.2	4	3.5	4.7	6	6	5.6	0	1.4
2.4	4.4	4.5	5.3	5.3	1.4	2.4	4.7	4	2.8	3.5	4.5	2.4
3.5	5.6	6.5	6	5.3	1.4	0	1.4	3.5	5.3	5.9	5.6	2.4
3.6	4.5	3.5	2.4	4.4	4.7	2.8	4	5.9	6.3	5.2	4.4	2.4
1.4	0	2.2	5.3	5.6	5	2.8	4	3.5	2.2	1.4	4.4	4
2.4	3.6	6.3	5.9	4	2.8	4.4	3.5	2.2	0	5.6	5.3	4.4
4.4	5.2	2.4	1.4	2.2	5	3.6	3.5	4	1.4	2.2	3.6	4.4
4.5	5.3	5.3	5.6	0	1.4	2.4	3.5	2.4	4	4.7	4.5	1.4
2.8	3.5	2.4	2.2	1.4	2.4	3.5	5.3	5.9	5.6	5.3	1.4	0
1.4	4.4	3.6	3.5	4.5	4.7	2.4	2.4	4	2.8	3.5	2.4	1.4
2.2	5.3	5.6	5	4.4	2.4	1.4	0	4.4	4	2.2	3.5	4
2.8	1.4	3.6	2.4	4	5.2	5.2	5	1.4	2.4	2.8	4.4	3.5
4.4	4.4	0	1.4	4	3.6	2.4	2.2	3.5	1.4	2.2	2.8	4.4

Table 1. (Continued)

5	5.2	2.4	3.6	4	5.2	2.4	3.6	4	1.4	0	4.4	4.4
3.5	1.4	3.5	2.2	2.2	5	4.4	2.2	1.4	4.7	4.5	3.5	2.4
4	3.5	2.2	4	4.4	0	1.4	2.4	3.6	1.4	2.8	2.4	5.6
5.3	3.5	2.4	4	3.5	2.2	1.4	4.7	4.5	3.5	3.6	4.4	1.4
0	1.4	4	2.4	2.4	2.8	5.3	5.6	4.4	3.6	2.8	2.4	1.4
2.2	4.5	4.7	4	2.4	3.5	2.4	1.4	0	3.5	2.8	1.4	2.4
2.4	3.5	4	4.4	3.5	4.4	4.4	5	1.4	2.4	2.8	2.2	1.4
3.6	4	3.5	0	2.4	2.2	1.4	4	3.6	2.2	2.2	4.5	4.7
4	3.6	2.8	2.4	1.4	3.5	3.5	1.4	2.4	2.8	2.4	0	2.4
1.4	4.4	5	4.4	4	2.4	2.8	2.4	3.5	3.5	4	3.6	1.4
2.2	2.8	2.4	1.4	2.2	2.4	0	1.4	3.6	4	3.5	3.5	3.5
4	3.6	4	2.4	2.8	2.4	2.2	2.2	2.4	2.8	2.4	1.4	1.4
5.6	4	4.4	5.2	4	5	6	6.8	3.5	4	1.4	0	